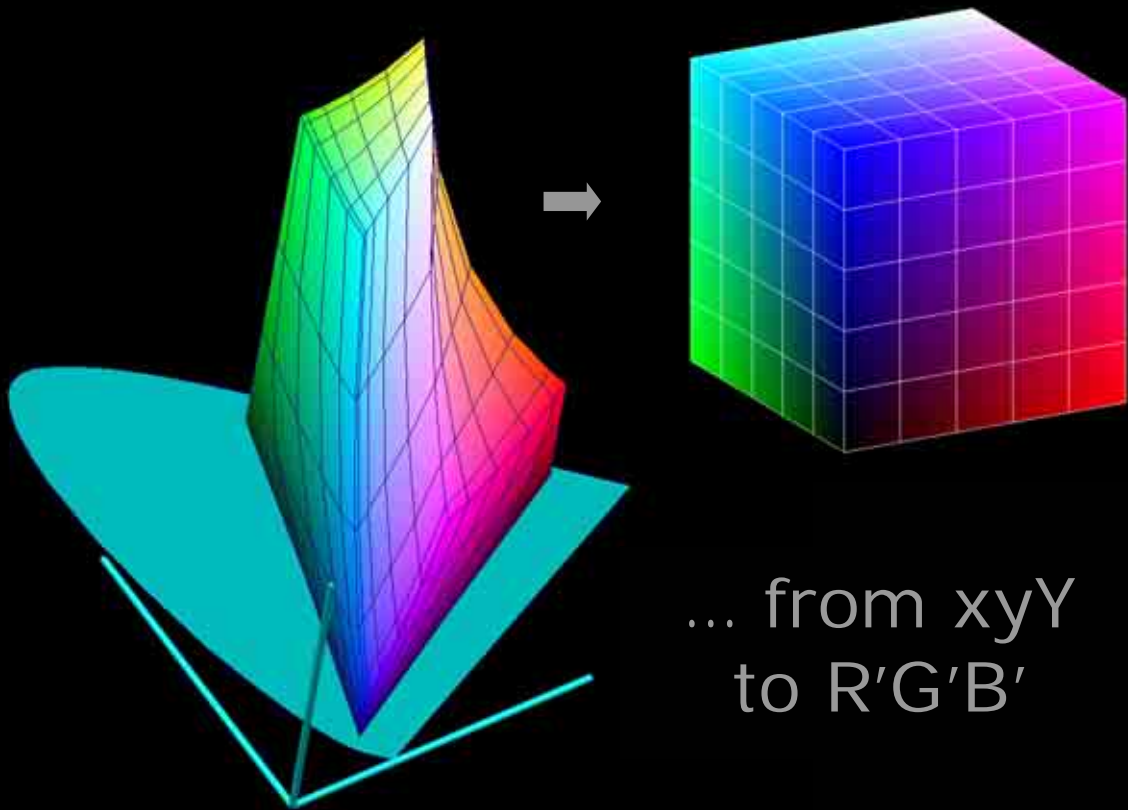




# A Review of RGB Color Spaces



... from  $xyY$   
to  $R'G'B'$

Danny Pascale

Title: A Review of RGB Color Spaces ...from xyY to R'G'B'

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Front cover: The xyY representation of the sRGB color space and its corresponding R'G'B' cube.

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Document revised 2003-10-06.

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**Why another document about RGB?** While there are many sources of information describing Red-Green-Blue spaces, their use, and why you should or should not use some of them, there are few self-contained sources of information on how to get there. You may find books, standards, and articles with equations on how to transform colorimetric data into a specific RGB space, or even how to translate data between some RGB spaces, but only a few spaces may be covered, or there is not enough information on how these formulas were derived and how to recalculate for different conditions, or they are not up to date with the most recent standards, or there are inaccuracies and mistakes, especially in non peer-reviewed freely accessible Web-based documents, or all of these “or”s.

**Why bother?** Accurate colorimetric data is no longer the realm of top tier professionals for which this is the only way to survive and which are willing, and capable, to invest heavily both time and money. Accurate colorimetric data formerly required proprietary – read expensive –, often esoteric, color management equipment and software. With the availability of low cost high quality devices such as scanners, printers, monitors, and calibration equipment, and with the significant increase in the per dollar computing power, accurate color capabilities are now available to small shops as well as consumers. The incentive to bother is that most users now expect a higher quality end product.

**Who should read this document?** This text is targeted to professionals who wish to acquire a basic understanding of colorimetry applied to computer and TV display systems and who want to see how theory translates into practice. A programmer involved in developing color transformation routines will find the flow chart and detailed conversion process of Section 4 helpful. Someone simply interested in checking the calibration of a camera or scanner using the GretagMacbeth ColorChecker card will find useful the RGB values of the card’s color patches presented, in Section 5, for many common spaces.

**What is not covered?** Although referred to in the text, ICC profile generation and gamut mapping are not developed. Nonetheless, this text is a good introduction to these more advanced topics.

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# 1 Introduction

Not so long ago, in fact just before the personal computers era, color displays used to be either color film (slides, prints and movies), the good old TV, or an image printed with ink on paper. These media were based on different color processes and standards, and the interchange between them, while done scientifically, involved complex dedicated machines and a good dose of black magic (some call it experience). The advent of the personal computer mixed the cards. Using an Apple Macintosh, anyone could soon do its own page layout. At first, the results were bland more often than not, and it proved once again that having a hammer does not make someone a carpenter. With time, the software evolved so that some graphic designer know-how is now integrated with the layout tools, in the form of Wizards – on screen step-by-step instructions – for example, and more time can be spent on the content than on the container.

The same situation is happening with color management. In the personal computer world, you used to have only one choice, the default Apple RGB space, whatever that was, since it was not really easy to find its specification. You could use it to generate images that would be compatible at least with future generations of Macintoshes. However, an image generated on a Macintosh did not look the same on a Windows based machine simply because, among other things, they do not have the same transfer function in the graphics card (one of the elements that control the display brightness in relation to graphic data), not to mention the limited amount of colors available on first generation Intel compatible computers. Later, proprietary color management schemes were imported to or created for the personal computer, and standards slowly emerged from them.

Very recently, the cross-platform acceptance of the International Color Consortium (ICC) color profiling method helped bring uniformity to the picture. It enables the input, output and display devices vendors to transparently, at least to the general user, exchange color data that conform to well characterized color spaces. The color management workflow tools are the latest trend in this development. In particular, the set of tools first offered by Adobe in their Photoshop version 5.0.2 program started a new era in the color environment controls available to the laymen. Not that the process is simple. There is a cumbersome heritage to support, to which we have to add all the proposed standards emerging from the work done on High-Definition TV (HDTV) and for the coming-real-soon-now computer-TV-multimedia convergence. The sheer number of combinations resulting from the various alternatives is frightening to the novice, annoying to the expert, and some of both for anyone in between. The fact that very good color output, either in print or electronically displayed, can now be obtained by a user who does not need to understand what makes the system work is a tribute to the skill of the programmers and engineers who designed these systems based on moving-sand standards, and to the complexity of the human eye and brain which marvelously adapt to a wide array of environmental conditions and compensate for many differences and errors in color reproduction.

The next section presents the fundamentals of color spaces based on the standards of the Commission Internationale de l'Éclairage (CIE), which are derived from the human eye response, and the methods used to assign numbers – color coordinates – to colors. The effects of the illuminant on the perceived color, and of the various non-linear compression or expansion operations – i.e. the “gammas” – on the recorded coordinates, are discussed. From there, the principal RGB color spaces that can be found are presented and defined. Follows a commented listing of key TV and multimedia standards that have some relevance to colorimetry. The procedure and equations required to convert colorimetric data to RGB data in device specific configurations are then detailed. Finally, as an example, R'G'B' values for the GretagMacbeth ColorChecker card color patches are given for the RGB color spaces defined previously.

## 2 Color spaces

Color models, like all mathematical representations of physical phenomena, can be expressed in many different ways, each with its advantages and drawbacks. Some representation are formulated to help humans select colors – the Munsell system for example – and others are formulated to ease data processing in machines, with the various RGB spaces all falling in this last category. The goal is to minimize formulation complexity and the number of variables while maximizing “substance” and breath of coverage. One thing they have in common is the number of variables, or dimensions. Historically, whatever the meaning assigned to the variables, three of them were enough to describe all colors: RGB, Hue-Saturation-Brightness (HSB) and other HS based models,  $L^*a^*b^*$ ,  $xyY$ , etc. From this observation alone, one would be tempted to conclude that color is perceived with a three-signal-output mechanism to the brain since nature often uses a minimalist approach to do things.

In many cases, more variables are added to complete a theory’s coverage or to supplement a physical limitation of the reproduction process. For example, black content (“K”) is added to cyan, magenta and yellow (CMY) inks to obtain better dark tones in traditional printing. Printing processes with more than four colors – Pantone Hexachrome with six colors, HiFi color with up to eight colors – have been developed to extend the reproducible color range. Some desktop printers are now offered with two additional color cartridges, consisting of light cyan and light magenta, which are designed to improve color gradients uniformity in the highlights, where print density would be low, and the dots visible, for normally concentrated cyan and magenta inks. These added variables are not additional dimensions per se since they are not totally independent of the primary coordinates – i.e. some of the colors generated by mixing the additional inks with the primaries can also be generated by mixing only the original primaries.

The first major distinction between color-spaces is device dependency. Color coordinates from a *device independent* space are the same on all output media. For example, discounting surface reflection effects such as “shininess” or gloss, if the coordinates of a car color and the coordinates of that color in an image of that car are the same, then these colors are identically perceived by the human eye. The  $xyY$  space, and its equivalent XYZ representation, falls in this category. Expressing the previous example differently, a stimulus characterized by a given XYZ triad will be perceived as the same color as a stimulus from another source which has the same XYZ triad. The XYZ space is based on how human perceive light and is thus independent of the media on which the color is seen. On the other hand, a *device dependent* color space will have different coordinates for the same color for various output media. All RGB and CMYK spaces fall in the device dependent camp since they are defined in relation to very specific primary colors, either phosphors or ink. As many have found, an RGB triad from an Apple Macintosh computer does not represent the same color as the triad with identical values on a Windows machine.

Another distinction is the ease with which the coordinates can be mentally associated with the codified color. The HSB space is being promoted as user-friendly since it is “relatively” easy to relate a HSB triad to the color represented by a given hue (the chromatic content, the presence of a color), its saturation (the ratio between chromatic and achromatic – i.e. white, gray, black, contents), and brightness (a relative-to-white lightness-darkness level; note: this is not the “accepted” definition of brightness, but the one used in the HSB model). This ease of use is also true for other hue-saturation based models but the varieties of name and definitions for the third parameter, brightness, value, or intensity, can bring confusion. On the other hand, a set of RGB or CMYK coordinates can be difficult to visualize, and  $xyY$  values are almost impossible to deal with for the infrequent user.

This being said, the most commonly used systems for exact color data interchange are nonetheless the  $xyY$  space, and its  $L^*a^*b^*$  derivative. The RGB space, which is used in most computer generated images, was not omitted for the pleasure of it. The reason is simply the presence of the word “exact” before the word “color” in the first sentence of this paragraph. There is a long tradition of using the term “RGB” without any mention of the environment – the display, operating systems, software or printer – used to generate the image, with the result that RGB is far from being a standard. Thankfully, there are rigorous ways of going from  $xyY$  coordinates to device specific RGB coordinates, and vice-versa.

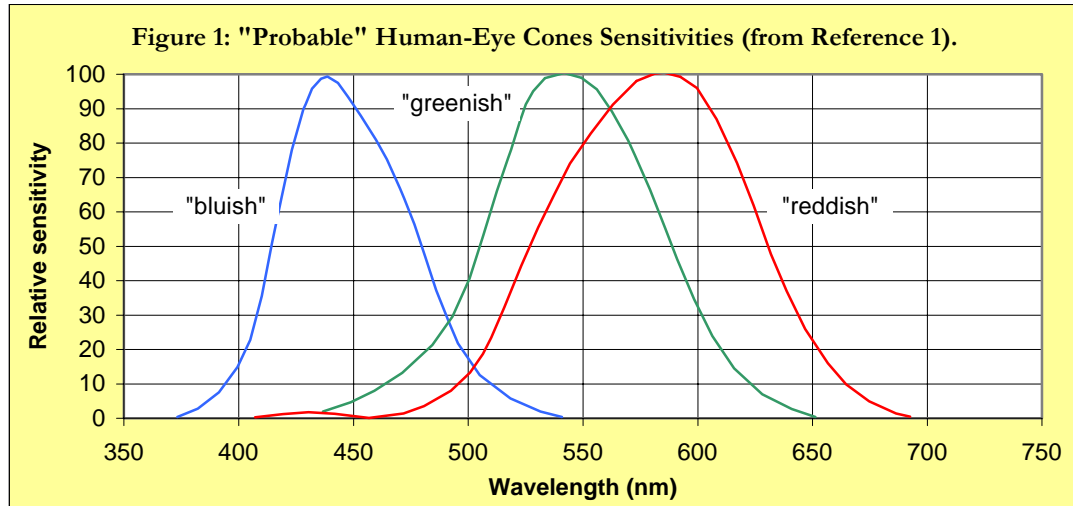
As if it was not already confusing, there is still another element in the recipe: *gamma*. Gamma is a way of mathematically expressing the non-linear perception of the eye to light intensity. It is also a way of expressing the relation between the light output of a monitor and the input voltage. It can even be a mathematical method to encode the data representing color so that more dynamic range in intensity is represented with fewer bits. In fact, all of these flavors of gamma have to be considered in understanding computer graphics.

## 2.1 Parameters of color spaces

### 2.1.1 The human eye

Color perception is a brain process that starts in the eye's cone receptors. These receptors are found in three varieties that exhibit somewhat reddish, greenish, and bluish sensitivities. The "probable" sensitivities, according to Hunt,<sup>1</sup> can be seen in Figure 1. The separation between colors is not clean and there is significant overlap between the sensitivities of all three varieties, particularly between the red and green cones.

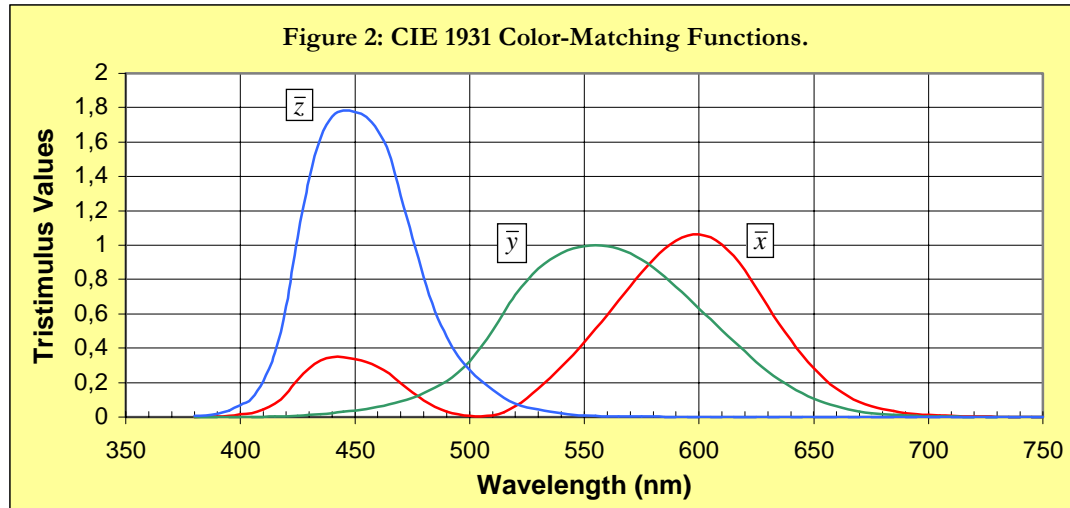
The other eye receptors, the rods, which are good at detecting luminance, but not color, in low light conditions, are saturated at the light levels relevant to the applications concerned by this text.



The experiments that brought us these curves were performed in the last 50 years whereas the *trichromatic theory* itself dates back to the seventeenth and eighteenth centuries from work done by Isaac Newton, Thomas Young, John Dalton, and others.<sup>2</sup> It is perhaps superfluous to mention that the modern measurements did bring some comfort to the practitioners and solidified the trichromatic theory base. We may be tempted to conclude from these findings that three chromatic signals go from the eye to the brain. However, some processing occurs near the light receptors and one current theory is that three signals, consisting of a lightness level plus two color difference signals, are transmitted, which is, as we surmised in this section's introduction, a number of signals equal to the number of variables of most color models. These color difference signals are analogous to the color encoding found in many television standards.

Figure 1 is crucial in understanding that any trichromatic theory cannot generate all the colors perceived by the brain, which is to say that, like most theories, there are some limitations. For example, using a modern laser-display system with monochromatic primaries at 635 nm (red), 532 nm (green), and 447 nm (blue), lets see if we can simulate the perception of a monochromatic light at 580 nm (an orange color). Since the monochromatic orange stimulus excites the greenish and reddish cones, a contribution is required by both the green and red primaries, while no contribution is required from the blue primary. The problem is that the green primary also excites the bluish cones, making it impossible to exactly replicate the orange stimulus. This situation is not exclusive to the set of primaries in this example and is mostly perceptible when trying to reproduce very pure or monochromatic colors.

So why should we use the trichromatic theory anyway? Because it works very well for most colors that are not monochromatic, and because there is no urgent need to add a fourth or fifth color stimulus to the picture. Not to mention the fact that monochromatic colors, typically generated by lasers or far away cosmic phenomenon, are seldom seen in our daily lives. Nonetheless, it can be shown that, in some instances, some of the out-of-range colors can be obtained by clever signal processing (see chapter 19 of Reference 1 for more information on this subject).



In 1931, the Commission Internationale de l'Éclairage (CIE) established standards for color representation based on the physiological perception of light. They are built on a set of three *color-matching functions*, collectively called the *Standard Observer*, related to the red, green and blue cones in the eye.<sup>3</sup> These functions are shown in Figure 2. They were derived by showing subjects color patches and asking them to match the color by adjusting the output of three pure (monochromatic) colors (435,8, 546,1, and 700 nm).

But then, how can it be reconciled that no trichromatic theory can explain all colors and that three primaries were used to build a space for all colors? The answer lies partly in Figure 1 and partly in a clever experimental procedure. In fact, it was possible to simulate all colors perceived by the eye by mixing the three primaries together AND by illuminating the reference color patches with some of the primaries. The amounts of each primary in the simulated patch and the compensating illumination on the reference patch were then processed to obtain the color-matching functions of Figure 2. It is important to note that the primaries referred to by the color-matching functions of Figure 2 are not the “real” primaries used in the experiments but mathematically derived “imaginary” versions of them which make it possible to cover the complete spectrum (those interested in knowing how they were derived should read chapters 7 and 8 of Reference 1).

The color-matching functions give the amount of each primary, called the *tristimulus value*, necessary to match a hypothetical equal-energy spectrum – same energy at all wavelengths, illuminant “E” in Table 3 – which would be seen as white. These functions are the basis of almost all modern color models. The Munsell system, which predates the CIE 1931 model and which is still in use, is one exception. It has its own reference frame made of carefully manufactured and controlled color patches designed for uniform perceptual differences between them, and to which unknown colors are compared. When required, this model can be translated to the CIE model.

### 2.1.2 The “abc” of XYZ

Color is not an intrinsic property of an object. It is the perception of the energy emitted or reflected from the object, once processed by the human visual system and the brain, which makes us assign colors to this energy. This psychophysical process is described by the color-matching functions of Figure 2. The mathematical derivation of color coordinates from these functions first requires measuring the spectrum of the source. This spectrum is usually expressed in terms of a Spectral Power Density (SPD), in Watt/nm, and it can be determined by separating the visible spectrum in a number of wavelength bands, with 10 nm per band for example, and by measuring the power, in Watt, within each band.

For reflected light, the reflectance is measured for each wavelength band and a ratio is computed relative to a perfect white diffuser. For transmitted light, a ratio is computed relative to a perfect transmitter. In the case of self-luminous sources, a radiance factor, the ratio between actual output and maximum output, may be determined.

Then, for each wavelength band, the reflectance, transmittance, or radiance factor is multiplied by the source SPD and by the spectral tristimulus value of each color-matching function. Results are then added separately for each



function. In other words, the reflected or transmitted spectrum is weighted by the color-matching functions and integrated to provide a single value, a scalar, also called the tristimulus value, for each function.

The scalar obtained with the  $\bar{x}$  color-matching function is named “X”. Similarly, the scalars obtained with the other functions are “Y” and “Z”.

Modern instruments automate this procedure, and the only task left to the operator is to point the probe toward the source or sample and press a button, which is not very helpful to learn the principle behind it. In practice, even with a very basic spectrometer, the computation is simple enough that it can be done manually. Spectral resolution does not need to be fine since the cone’s response curve is very broad and smooth, and measurements can be done at every 5, 10 or 20 nm. The only important point is that the data be measured with a *radiometric* detector, which means an instrument with uniform sensitivity across the spectrum. Radiometric units are based on the Watt, but in practice, since the data is often normalized, any detector output such as current or voltage can be used directly, as long as the detector is characterized in terms of signal response, ideally linear, and spectral sensitivity, ideally uniform. A *photometric* detector should not be used. Photometric detectors are based on the lumen, and its derivative units: lux, lambert, candela, nit, etc., which take into consideration the overall spectral response of the human eye, with its maximum sensitivity in the green portion of the spectrum and decreasing sensitivities going into the red or blue regions.

Once the spectral reflectance or transmittance data is gathered, it is processed in the fashion described by Table 1. For each wavelength step the reflectance, transmittance, or radiance factor (column B), expressed as a value between zero and one, is multiplied by the source SPD (C). The source SPD may not have to be measured if the source is one of the many standard illuminants for which tabulated data is available. The data is further multiplied by the corresponding spectral tristimulus value of each color-matching function (D1, D2 or D3), as well as by the wavelength step (E). Intermediary results (F1, F2, or F3) are then summed over all steps – i.e. numerically integrated.

An additional calculation (G) is performed for calibration purposes. The source SPD multiplied by the  $\bar{y}$  color-matching function is integrated to provide the calibration constant “k”. The  $\bar{y}$  color-matching function was defined in such a way that it matches the spectral response of the human eye (X and Z have no such easily attributed correspondence to a real phenomenon). “k” is thus, by definition, a photometric quantity and so are all values represented by Y. Y is the reflectance, transmittance, or radiance weighted by the eye sensitivity and is equal to 100 when the reflectance, transmittance, or radiance are equal to one for all wavelengths. Therefore, the color coordinates of the source are, by definition, the ones for which Y equals 100.

(A) wavelength (nm)	(B) Reflectance, Transmittance, or Radiance (%)	(C) source Spectral Power Density (SPD) (« power » / nm)	(D) CIE 1931 spectral tristimulus values			(E) wavelength step (nm)	(F) = (B) x (C) x (D_) x (E)  (« power »)			(G) = (C) x (D2) x (E) (used for calibration) (« power »)	
			(D1)	(D2)	(D3)		(F1)	(F2)	(F3)		
			$\bar{x}(\lambda)$	$\bar{y}(\lambda)$	$\bar{z}(\lambda)$						
380			0,0014	0	0,0065	5					
385			0,0022	0,0001	0,0105	5					
...			...	...	...	5					
585			0,9786	0,8163	0,0014	5					
...			...	...	...	5					
775			0,0001	0	0	5					
780			0	0	0	5					
<b>SUM :</b>											
<b>SUM x (100 / k) :</b>											
=							<b>X</b>	<b>Y</b>	<b>Z</b>		<b>= k</b>

Table 1: A detailed method to determine XYZ tristimulus values from measured data. Derived from ASTM E308-99.

(A) wavelength (nm)	(B) Reflectance (%)	(D) D65 source SPD x CIE 1931 tristimulus values			(F) = (B) x (D <sub>-</sub> )  (« power »)		
		(D1)	(D2)	(D3)	(F1)	(F2)	(F3)
		W <sub>x</sub>	W <sub>y</sub>	W <sub>z</sub>			
400 -	0,3483	0,121	0,003	0,575	0,042	0,001	0,200
410	0,4273	0,311	0,009	1,477	0,133	0,004	0,631
420	0,4563	1,164	0,033	5,581	0,531	0,015	2,547
430	0,45678	2,400	0,092	11,684	1,096	0,042	5,337
440	0,44942	3,506	0,221	17,532	1,576	0,099	7,879
450	0,43662	3,755	0,413	19,729	1,640	0,180	8,614
460	0,42362	3,298	0,662	18,921	1,397	0,280	8,015
470	0,40476	2,141	0,973	14,161	0,867	0,394	5,732
480	0,38328	1,001	1,509	8,730	0,384	0,578	3,346
490	0,36026	0,293	2,107	4,623	0,106	0,759	1,665
500	0,33704	0,028	3,288	2,769	0,009	1,108	0,933
510	0,31172	0,054	5,122	1,584	0,017	1,597	0,494
520	0,26668	0,581	7,082	0,736	0,155	1,889	0,196
530	0,22506	1,668	8,833	0,421	0,375	1,988	0,095
540	0,20396	2,860	9,472	0,191	0,583	1,932	0,039
550	0,2021	4,257	9,830	0,081	0,860	1,987	0,016
560	0,19362	5,632	9,446	0,034	1,090	1,829	0,007
570	0,18848	6,960	8,709	0,018	1,312	1,641	0,003
580	0,19974	8,344	7,901	0,015	1,667	1,578	0,003
590	0,21676	8,676	6,357	0,009	1,881	1,378	0,002
600	0,22714	9,120	5,379	0,007	2,072	1,222	0,002
610	0,23448	8,568	4,259	0,003	2,009	0,999	0,001
620	0,2325	7,119	3,149	0,001	1,655	0,732	0
630	0,23366	5,049	2,070	0	1,180	0,484	0
640	0,25284	3,522	1,370	0	0,891	0,346	0
650	0,29694	2,112	0,794	0	0,627	0,236	0
660	0,35894	1,229	0,454	0	0,441	0,163	0
670	0,41466	0,658	0,240	0	0,273	0,100	0
680	0,44788	0,331	0,120	0	0,148	0,054	0
690	0,46516	0,142	0,051	0	0,066	0,024	0
700 +	0,47134	0,147	0,053	0	0,069	0,025	0
<b>SUM :</b>					<b>25,15</b>	<b>23,66</b>	<b>45,76</b>
<b>=</b>					<b>X</b>	<b>Y</b>	<b>Z</b>

Table 2: A simplified method to determine XYZ tristimulus values from measured data applied to reflectance measurements of the GretagMachbeth ColorChecker card “Blue Flower” sample, with a D65 Illuminant. As per ASTM E308-99, when spectral data is not available, the weights of the color-matching functions between 360 nm and 390 nm were added to the 400 nm weight, and the weights of the color-matching functions between 710 nm and 780 nm were added to the 700 nm weight.

If the source SPD was determined in absolute radiometric units, then “k” represents an absolute photometric reference. If this was not the case, an absolute photometric reference can still be obtained by measuring the maximum output of a self-luminous source, or the reflection of a perfectly diffusing sample, with a photometer. For example, an 82 cd/m<sup>2</sup> luminance is typical of what can be measured with a photometer on a modern Cathode Ray Tube (CRT). The computed Y values of all measured colors obtained from spectral measurements would then be scaled to this luminance to obtain absolute luminance data, Y = 100 corresponding to 82 cd/m<sup>2</sup>, and so on.

An excellent complete source of data, presented in tabular forms, on the SPD of all standard illuminants, such as C, D50, D65, F6 etc., which are presented in more details in Section 2.1.4, and for the color-matching functions, is ASTM Standard E308-99<sup>4</sup> which describes the procedure of Table 1 in detail as well as a simplified procedure that can be used with standard sources. The simplified procedure is demonstrated in Table 2 where the XYZ coordinates of the “Blue Flower” sample<sup>5</sup> found in the GretagMacbeth ColorChecker<sup>6</sup> card are computed for the D65 illuminant, which is a standardized version of typical North Sky Daylight.

In Table 2, the source SPD data is already combined with the spectral tristimulus data. This data is available in tabular forms in ASTM E308-99 for both 10 nm and 20 nm steps, and for most standard illuminants. Also, while the color-matching functions are defined in the 360 to 830 nm range, it is not always possible or necessary to gather data within this range. Using only the 380 to 780 nm range will not lead to significant error in most cases since the color-matching functions weights are very small outside of it. On the other hand, if data is not available for even the reduced 380 to 780 nm range, as is the case in Table 2, a procedure is suggested in ASTM E308-99 to fill-in the blanks.

The procedure calls for adding the weights of all the color-matching functions for which there is no data to the first wavelength where data is available. In Table 2, this means adding the weights of the color-matching functions between 360 nm and 390 nm to the 400 nm weight, and adding the weights of the color-matching functions between 710 nm and 780 nm to the 700 nm weight.

With D65 as an illuminant, the sample has XYZ coordinates of (25,15 23,66 45,76)<sub>D65</sub>. A similar calculation with D50, an illuminant representative of high power tungsten lighting, gives XYZ coordinates of (24,64 23,41 34,43)<sub>D50</sub>. Comparing the data sets we see that the sample under the D50 illuminant has significantly less bluish content, the Z coordinate, a result which is coherent with the expectation of more reddish-orange content from tungsten illumination but not necessarily intuitive to deduce.

Another way of presenting the tristimulus data is to determine the proportion of each value relative to the sum of all three. These ratios are defined as:

$$x = \frac{X}{(X+Y+Z)} \quad y = \frac{Y}{(X+Y+Z)} \quad z = \frac{Z}{(X+Y+Z)} \quad , \quad (1)$$

with, as a complement, the relation

$$x+y+z=1 \quad . \quad (2)$$

The xyz coordinates of the Blue Flower sample are (0,266 0,250 0,484)<sub>D65</sub> and (0,299 0,284 0,417)<sub>D50</sub>. Again, comparing the two sets, we see an increase in the reddish-greenish content (x and y), or more yellow, and a decrease of the bluish content, a more intuitive result than the one deduced from XYZ data.

D50, D65 and all other so-called white illuminants have hues that are not perceived when the illuminant is used alone. The eye, with help from the brain, adapts itself to make these illuminants look like what we expect: pure white. This adaptation to white is effective for a certain range of chromaticities only and, for example, CIE ILL A, a low power incandescent type illuminant, will always have a faint yellowish orange hue.

In the xyz representation, because of the redundancy of Equation (2), only two coordinates are required, usually “x” and “y”, to convey the chromatic content of a sample. The representation of color is thus simplified from 3 dimensions to 2 dimensions. However, the absolute luminance information of Y is lost in the process. For these reasons, it is a common practice to present color data as xyY.

When plotted in the xy notation, the pure monochromatic colors of the spectrum form the now familiar horseshoe shape of Figure 3. The straight line at the base of the horseshoe represents the mixture of red and blue light, two

colors at the opposite of the spectrum. All other “impure”, or non-monochromatic, or less saturated colors fall within the horseshoe. Only the colors inscribed within the horseshoe are possible. The colors outside the horseshoe are “imaginary” and result from the mathematical treatment behind the color-matching functions. The horse shoe is inscribed in a larger triangle, defined by the  $(x,y) = (0,0)$ ,  $(1,0)$ , and  $(0,1)$  coordinates, which is called the Maxwell triangle, from the name of the Scottish physicist, James Clerk Maxwell (1831-1879), who used a similar triangle to understand color and which is considered the inventor of the trichromatic photographic process.

The more one goes away from the edges of the horseshoe, the more the color is de-saturated. The ideal white, also called the *equal energy illuminant* since all three reference functions are equal, has  $x$ ,  $y$  and  $z$  equal to 0,33333... and is located somewhere in the center of the horseshoe. It is interesting to note that the ideal black is located at the same spot. This seemingly contradictory result is simply because the diagram does not represent intensity, thus its name *chromaticity diagram*, and the importance of the “Y” information in comparing measured color data.

A very useful feature of this diagram is that it can be used to determine the color resulting from the mix of two known emissive colors. The chromaticity of a color resulting from the mixture of two colored lights will simply be located on the straight line between the two. This is one of the characteristics of additive color mixture, also called *Grassmann's laws*. Adjusting the ratio between the two lights will make the resulting color move along the line. An interesting consequence is that mixing two colors located at such positions on the chromaticity diagram that a line between them goes through the white point region will result in “white” being perceived for certain ratios. This last example is just to contradict the often-heard statement that you need at least three colors to generate white, and is a direct consequence of the overlapping bandwidths of the cones.

Similarly to the color mix obtained with two sources, we can extend the concept to three sources. To be called primaries these sources have to be selected in such a way that it is not feasible to simulate one of them by mixing the two others. From this requirement we see that three sources will enclose a triangular shape. Mixing the primaries in various proportions will generate all the colors within the triangle, also called the *color gamut*. This property of the diagram makes it easy to understand how color TV and computer monitors use only three different phosphors to simulate a multitude of colors.

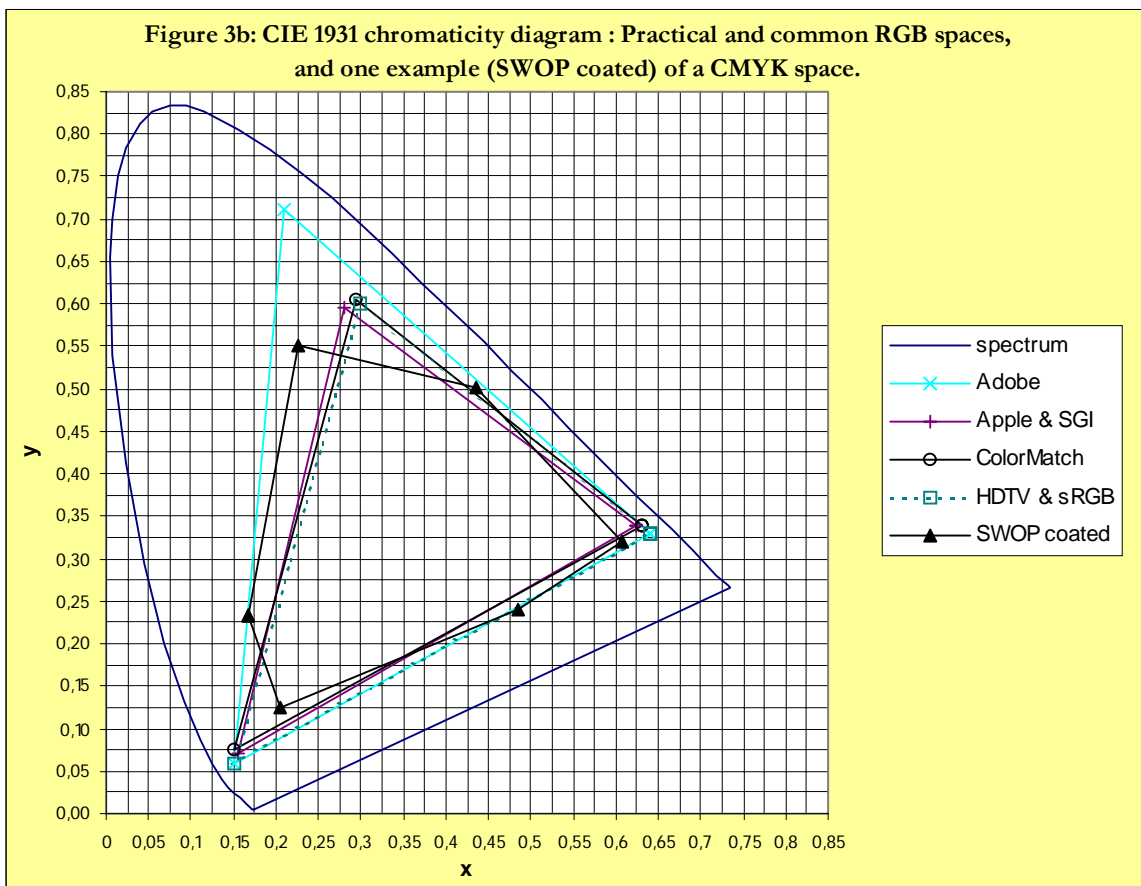
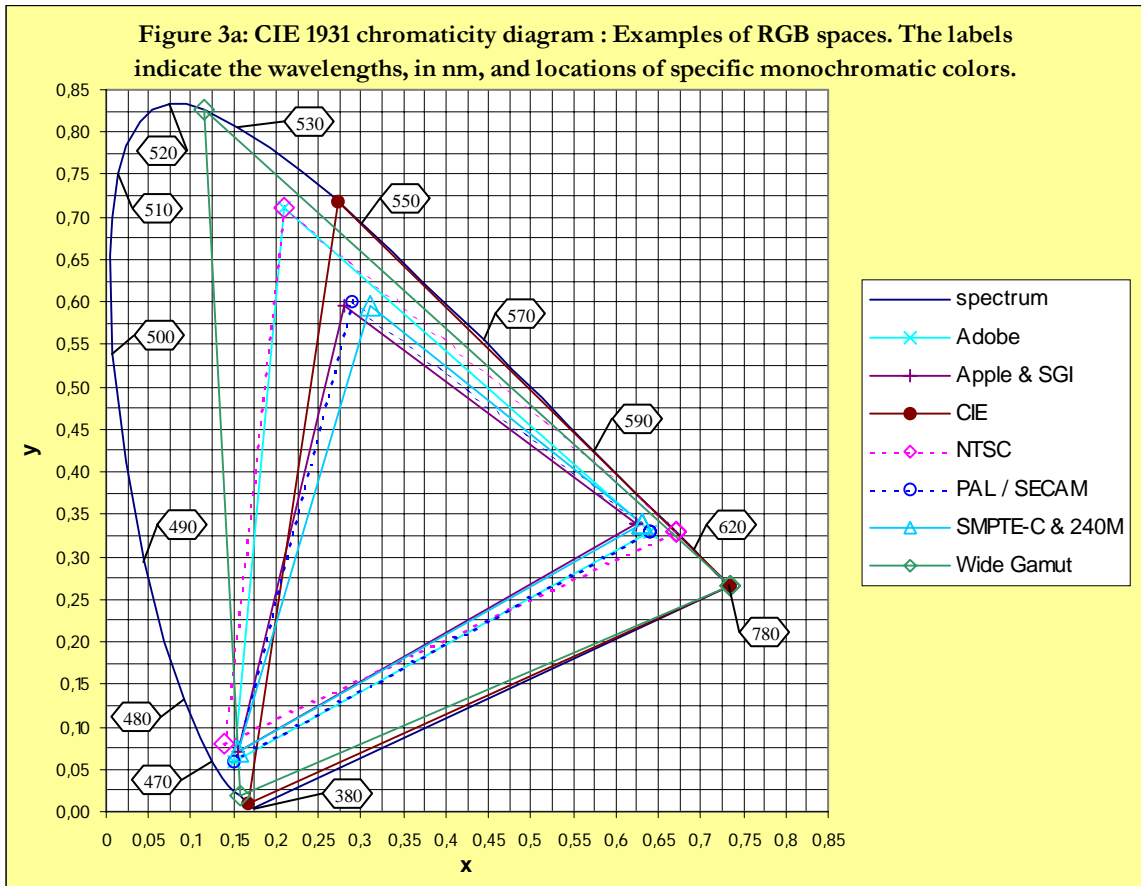
One of the first challenges is to select the best primaries to generate a maximum number of colors. As we saw before, it is impossible to generate all colors with three primaries. Ideally, you could use four, five, or more, different basic colors to define a multi-facet polygon that would encompass most of the horseshoe shape. This is difficult in practice for a CRT display, firstly because of the limited availability of high-brightness long-life monochromatic phosphors – quite a challenge in itself –, and secondly because of the complexity of controlling multiple partially color-redundant sources, a technology which is presently not cost-effective for a consumer level product. The phosphors of the first color TVs were selected in most part for the two following reasons: they were available, and three phosphors are enough to get a very good job done.

According to a study that looked at how to optimize the luminous efficiency of displays with three primaries, the ideal wavelengths are 610, 530, and 450 nm.<sup>7</sup> The rationale here is that, as long as the cost per watt for monochromatic light is the same for all wavelengths, the system cost is driven by the eye sensitivity. For example, for a given brightness sensation, significantly more power is required at 410 nm than at 450 nm. Practical laser display systems have been developed with 635, 532, and 447 nm primaries,<sup>8</sup> and with 656, 532, and 457 nm primaries.<sup>9</sup>

Figure 3a shows various RGB spaces proposed in the last decades. Figure 3b shows practical and common contemporary choices. All of these are discussed in more details in Section 2.2.

Figure 3b also contains the gamut for the “SWOP coated” printing process (SWOP: Specifications for Web Offset Publications) a widely used process based on CMYK ink primaries. Contrary to monitor displays that generate color by adding lights together, called an additive process, printing is a subtractive process. The subtractive process is not perfectly linear, mixing two inks, like mixing two paints, will not always give a color on a line between the originals. Some mixes will, and others will not. The gamut shown in Figure 3b has six apexes, one for each C, M and Y component, and one for the M+Y (or red), C+Y (or green), and C+M (or blue) components. In particular, we can see that the C+Y mix is far away from a straight line mix of C and Y and that the gamut is much bigger than what would have been deduced with the CMY primaries only. Also, the straight lines between the apexes are approximations of the mixtures between these points.

The representation of Figure 3 has many other practical applications that are not within the scope of this short tutorial. The reader with an affinity for applications of color science in art is encouraged to read the book by Agoston<sup>10</sup> for an in-depth presentation of the subject.



### 2.1.3 Limitations of the CIE 1931 chromaticity diagram

As good as it may be, the CIE 1931 chromaticity diagram presented in the preceding section is not without faults. Soon after it was issued, it was found that it does not represent color gradations in a uniform matter. David L. MacAdam<sup>11</sup> showed in the early forties that the minimum distance between two discernible colors is smaller in the lower left portion of the horseshoe, and progressively bigger toward the top. It is also non-uniform; for any given point, tracing the coordinates of the minimally discernible colors around the point forms an ellipse, called a *MacAdam ellipse*.

Attempts to transform the original diagram into a more uniform representation have first resulted in the 1960 CIE Uniform Chromaticity Space (UCS), where projective transforms of the x and y space distort it to obtain somewhat more uniform u and v coordinates. More recently, there was industry wide “agreement” on two standards, the L\*a\*b\* representation – called either CIE 1976 (L\*a\*b\*) or CIELAB – and the L\*u\*v\* space – called either CIE 1976 (L\*u\*v\*) or CIELUV –, the later a slightly revised version of the 1960 CIE UCS. Since both spaces have their proponent and preferred applications, it is up to the users to select the most appropriate model, at least until a better “universal” one is defined and accepted. The L\*a\*b\* space will be used in this document.

The L\*a\*b\* is derived from the XYZ data with the following transform:

$$\begin{aligned}
 L^* &= 116(Y/Y_n)^{1/3} - 16 && \text{(for } Y/Y_n > 0,008856) \\
 L^* &= 903,3(Y/Y_n) && \text{(for } Y/Y_n \leq 0,008856) \\
 a^* &= 500\left(\left(X/X_n\right)^{1/3} - \left(Y/Y_n\right)^{1/3}\right) \\
 b^* &= 200\left(\left(Y/Y_n\right)^{1/3} - \left(Z/Z_n\right)^{1/3}\right)
 \end{aligned}
 \tag{3}$$

where  $X_n$ ,  $Y_n$ ,  $Z_n$ , are the values of X, Y, Z for a specified *reference white* – i.e. illuminant. Better uniformity is thus obtained by normalizing the color coordinates by the illuminant coordinates, and by applying a 1/3 exponent to the ratios, which corresponds to the non-linear perception – i.e. dynamic range compression – of the eye subjected to increased luminance.

The relation for  $L^*$  for  $Y/Y_n$  ratios of less than 0,008856 has been presented for completeness, but the reader will realize that it corresponds to quite dark colors.

Another limitation of the CIE 1931 representation is that it was determined from color patches covering a two degrees Field Of View (FOV). This FOV is well within the angle subtended by the eye’s fovea, the region of the retina near the eye’s optical axis where the density of cones is the highest. Cone density falls rapidly to less than ten times the peak value at plus or minus five degrees from the fovea center<sup>12</sup> and, in practice, color patches subtending FOVs between one and four degrees can be treated using the CIE 1931 data. For larger patches it was found that the eye has a somewhat different response. This resulted in a new set of measurements called the CIE 1964 data set that was done for patches subtending a FOV of ten degrees. Data corresponding to the CIE 1964 data set is presented as  $(X_{10}, Y_{10}, Z_{10})$  or  $(x_{10}, y_{10}, Y_{10})$  to distinguish it from the 1931 system.

Since most displays and print material are made of combinations of small color patches, the CIE 1931 system remains the system of choice for this analysis.

## 2.1.4 Illuminants

Illuminants, either standard or custom, cannot be dissociated from the XYZ data they helped generate. When providing colorimetric data, information on the illuminant used for the measurements always has to be given in order to understand and further process the data. Various standard illuminants have been devised to satisfy the evolving needs. Table 3 shows the coordinates of the principal standard illuminants in the CIE 1931 and CIE 1964 systems.

Illuminant	Description	CIE 1931					CIE 1964				
		x	y	z	X	Z	x	y	z	X	Z
A	Tungsten or Incandescent, 2856 K	0,44757	0,40744	0,14499	109,850	35,585	0,45117	0,40594	0,14289	111,144	35,200
B	Direct Sunlight at Noon, 4874 K * (obsolete)	0,34840	0,35160	0,30000	99,090	85,324					
C	North Sky Daylight, 6774 K *	0,31006	0,31615	0,37379	98,074	118,232	0,31039	0,31905	0,37056	97,285	116,145
D50	Daylight, used for Color Rendering, 5000 K *	0,34567	0,35850	0,29583	96,422	82,521	0,34773	0,35952	0,29275	96,720	81,427
D55	Daylight, used for Photography, 5500 K *	0,33242	0,34743	0,32015	95,682	92,149	0,33411	0,34877	0,31712	95,799	90,926
D65	New version of North Sky Daylight, 6504 K *	0,31273	0,32902	0,35825	95,047	108,883	0,31382	0,33100	0,35518	94,811	107,304
D75	Daylight, 7500 K *	0,29902	0,31485	0,38613	94,972	122,638	0,29968	0,31740	0,38292	94,416	120,641
9300 K	High eff. blue phosphor monitors, 9300 K	0,28480	0,29320	0,42200	97,135	143,929					
E	Uniform energy Illuminant, 5400 K *	0,33333	0,33333	0,33334	100	100	0,33333	0,33333	0,33334	100	100
F2	Cool White Fluorescent (CWF), 4200 K *	0,37207	0,37512	0,25281	99,186	67,393	0,37928	0,36723	0,25349	103,279	69,027
F7	Broad-band Daylight Fluorescent, 6500 K *	0,31285	0,32918	0,35797	95,041	108,747	0,31565	0,32951	0,35484	95,792	107,686
F11	Narrow-band White Fluorescent, 4000 K *	0,38054	0,37691	0,24254	100,962	64,350	0,38543	0,37110	0,24347	103,863	65,607

Table 3: Coordinates of the principal standard illuminants in the CIE 1931 and CIE 1964 systems. Temperatures followed by “\*” are correlated color temperatures. “Y” coordinates are normalized to 100.

It is common to associate a temperature with an illuminant. This temperature is related to the emission of a *blackbody*. A blackbody is by definition a material that has perfect emissivity and absorptivity at all wavelengths – it will therefore not reflect or scatter light. The light emitted from the blackbody has a spectral content with a dominant color that shifts from red to blue with increasing material temperature. Temperature is expressed in the Kelvin scale, with zero Kelvin defined as the absolute zero (-273 Celsius). The spectral content is described by Planck’s radiation law while the peak of the spectral curve is described by Wien’s displacement law<sup>13</sup>:

$$\lambda_{\max} = 2897885 / T \text{ nm}, \quad (4)$$

with T expressed in Kelvin. As we shall see, the peak does not correspond exactly to the perceived dominant hue but is a good indication of where the dominant color lies in the spectrum.

Few illuminants are perfect blackbodies. However, when a source matches the chromaticity of a blackbody, we refer to the source temperature as the *color temperature*. If the chromaticity does not match, the blackbody temperature that most closely matches the spectral properties of the illuminant is given; this temperature is called the *correlated color temperature*.

The illuminant referred to as 9300 K in Table 3 has the same chromaticity as a blackbody. Its spectrum has a significant short wavelength content and  $\lambda_{\max}$  as determined with Equation (4) is 312 nm, way into the ultraviolet. The illuminant referred as D65, with a correlated temperature of 6504 K, emits light with a spectrum close to mid-day daylight illumination and can be considered a good “general use” white. At this temperature,  $\lambda_{\max}$  is 446 nm, a deep blue. D65 is part of the standard CIE D series illuminants which cover the 4000 K to 10000 K plus range where the number following the “D” is an abbreviation of the correlated temperature – all D series illuminant have chromaticities slightly different than same temperature blackbodies.

For D50, with a correlated 5000 K temperature, the spectrum has a strong orange content typical of tungsten lights and  $\lambda_{\max} = 580$  nm, a well-defined orange. D50 is the reference illuminant for the print industry and the only illuminant used to compute L\*a\*b\* data in Adobe Photoshop. D50 is also, presently, the only illuminant in the *Profile Connection Space* (PCS), a color space used as the link between devices, in the International Color Consortium (ICC) profile definition.<sup>14</sup>

### 2.1.5 The Bradford Matrix

Transforming colorimetric data taken with one illuminant into data corresponding to another illuminant is often required. In the ideal case, the spectral data of each color sample is reprocessed with the new illuminant using the method shown in Table 2. However, this computer intensive process is not efficient and requires a large spectral database for each color. But more importantly, for most applications, like image processing, spectral data is simply not available. All modern color appearance models competing for international acceptance<sup>15</sup> incorporate such a *chromatic adaptation transform*. One contender that has withstood critical review is called the Bradford, or BFD for short, chromatic adaptation transform, from the name of the city, in England, where the researchers who developed it came from.

A simplified matrix representation of the Bradford transform was found to give excellent results during the work performed in the development of the sRGB standard.<sup>16</sup> In its simplified version, the only data required to generate the Bradford matrix are the XYZ coordinates of the source and destination whites. The *source white* is the illuminant used to measure the original data, and the *destination white* is the illuminant to which the data has to be translated. The Bradford conversion matrix is derived with the following relations:

$$\begin{bmatrix} R_{dw} \\ G_{dw} \\ B_{dw} \end{bmatrix} = \begin{bmatrix} 0,8951 & 0,2664 & -0,1614 \\ -0,7502 & 1,7135 & 0,0367 \\ 0,0389 & -0,0685 & 1,0296 \end{bmatrix} \bullet \begin{bmatrix} X_{dw} \\ Y_{dw} \\ Z_{dw} \end{bmatrix} \quad (5)$$

$$\begin{bmatrix} R_{sw} \\ G_{sw} \\ B_{sw} \end{bmatrix} = \begin{bmatrix} 0,8951 & 0,2664 & -0,1614 \\ -0,7502 & 1,7135 & 0,0367 \\ 0,0389 & -0,0685 & 1,0296 \end{bmatrix} \bullet \begin{bmatrix} X_{sw} \\ Y_{sw} \\ Z_{sw} \end{bmatrix} \quad (6)$$

$$\begin{bmatrix} Bradford \\ 3 \times 3 \\ matrix \end{bmatrix} = \begin{bmatrix} 0,9870 & -0,1471 & 0,1600 \\ 0,4323 & 0,5184 & 0,0493 \\ -0,0085 & 0,0400 & 0,9685 \end{bmatrix} \bullet \begin{bmatrix} R_{dw} / R_{sw} & 0 & 0 \\ 0 & G_{dw} / G_{sw} & 0 \\ 0 & 0 & B_{dw} / B_{sw} \end{bmatrix} \bullet \begin{bmatrix} 0,8951 & 0,2664 & -0,1614 \\ -0,7502 & 1,7135 & 0,0367 \\ 0,0389 & -0,0685 & 1,0296 \end{bmatrix} \quad (7)$$

where  $(RGB)_{dw}$  and  $(XYZ)_{dw}$  are the coordinates of the destination white, and  $(RGB)_{sw}$  and  $(XYZ)_{sw}$  are the coordinates of the source white. In Equations (5), (6) and (7), the 3x3 matrix, with "0,8951" as its top-left element, is called the *cone response matrix*. In Equation (7), the 3x3 matrix, with "0,9870" as its top-left element, is called the *inverse cone response matrix*. These two matrices are, as their name says, the inverse of one another, and multiplying one by the other will result in a unitary diagonal matrix.

$(RGB)_{dw}$  and  $(RGB)_{sw}$  are first calculated with Equations (5) and (6), then the Bradford matrix is determined from Equation (7) using these values. Bradford matrices for specific sets of CIE 1931 illuminants, whose XYZ coordinates can be found in Table 3, are shown in Table 4.

Here is an example of the use of the Bradford matrix: Lets say you need to determine the XYZ coordinates of the ColorChecker "Blue Flower" sample as it would be measured with a D50 illuminant but you only have the XYZ coordinates obtained with the D65 illuminant, and no sample. The XYZ coordinates of this sample as seen with the D65 illuminant were shown to be (25,15 23,66 45,76)<sub>D65</sub> in Table 2. Using the D65-to-D50 Bradford matrix we obtain:

$$\begin{bmatrix} 24,60 \\ 23,40 \\ 34,54 \end{bmatrix}_{D50} = \begin{bmatrix} 1,0478 & 0,0229 & -0,0501 \\ 0,0295 & 0,9905 & -0,0171 \\ -0,0092 & 0,0150 & 0,7521 \end{bmatrix} \bullet \begin{bmatrix} 25,15 \\ 23,66 \\ 45,76 \end{bmatrix}_{D65} \quad (8)$$

The result is very close to what is computed for D50 with the more precise method of Table 2, (24,64 23,41 34,43)<sub>D50</sub>, and is indicative of the precision that can provide the simplified representation of the Bradford matrix.



<b>A --&gt; C</b>	<b>C --&gt; D50</b>	<b>D50 --&gt; D65</b>
0,8530 -0,1130 0,4404 -0,1239 1,0854 0,1426 0,0912 -0,1554 3,4776	1,0377 0,0154 -0,0583 0,0171 1,0057 -0,0189 -0,0120 0,0204 0,6906	0,9556 -0,0230 0,0632 -0,0283 1,0099 0,0210 0,0123 -0,0205 1,3299
<b>C --&gt; A</b>	<b>D50 --&gt; C</b>	<b>D65 --&gt; D50</b>
1,2040 0,1030 -0,1567 0,1407 0,9280 -0,0559 -0,0253 0,0388 0,2892	0,9649 -0,0164 0,0810 -0,0161 0,9941 0,0258 0,0173 -0,0297 1,4486	1,0478 0,0229 -0,0501 0,0295 0,9905 -0,0171 -0,0092 0,0150 0,7521
<b>A --&gt; D50</b>	<b>A --&gt; D65</b>	<b>C --&gt; D65</b>
0,8779 -0,0915 0,2566 -0,1117 1,0925 0,0852 0,0502 -0,0838 2,3994	0,8447 -0,1179 0,3948 -0,1366 1,1042 0,1292 0,0799 -0,1349 3,1924	0,9904 -0,0072 -0,0116 -0,0124 1,0156 -0,0029 -0,0036 0,0068 0,9182
<b>D50 --&gt; A</b>	<b>D65 --&gt; A</b>	<b>D65 --&gt; C</b>
1,1574 0,0872 -0,1269 0,1199 0,9219 -0,0456 -0,0200 0,0304 0,4178	1,2165 0,1110 -0,1549 0,1533 0,9152 -0,0560 -0,0239 0,0359 0,3148	1,0098 0,0070 0,0128 0,0123 0,9847 0,0033 0,0038 -0,0072 1,0892

Table 4: Bradford Matrices between various sets of standard CIE 1931 illuminants.

### 2.1.6 Gamma

The eye is more sensitive to variations of luminance in low luminance levels than similar variations in high luminance levels. R'G'B' values, commonly referred to by “RGB” in most application software, are scaled according to this non-linear perception of the eye and more data triads are assigned to the lower luminance levels. As a result, the R'G'B' scale is close to a perceptively linear scale where doubling the values of a triad will result in a color whose brightness appears doubled

Gamma ( $\gamma$ ) is a subject of much debate. Even the use of the word gamma is an element of discord. Originally coined to explain the straight-line portion of the S-shaped (sigmoid) curve obtained when tracing, on log-log scales, the optical density of photographic film in relation to exposure, the so-called H&D curve from its inventors Hurter and Driffield, it has been since misused and overused. Some authors, for the sake of scientific rectitude, even proscribe the use of gamma in relation to displays and propose the more generic term “exponent” instead. We will nonetheless continue to use the term gamma in this paper since it is associated with fundamental aspects of display technology and human perception, to which a generic term like “exponent” would not do justice. However, you should always verify how gamma is defined before making comparisons with other sources of information, and you should get used to the fact that any author’s gamma value could be the reciprocal of another author’s definition.

A very thorough presentation of modern CRT characteristics is contained in a paper by Berns, Motta and Gorzynski.<sup>17</sup> Easily readable presentations of gamma can be found in the book and the Internet articles of Poynton.<sup>18</sup> The definition of the various flavors of gamma is well presented in a tutorial that is part of the Portable Network Graphics (PNG) Specification<sup>19</sup> published by the World Wide Web Consortium (W3C). A typical vision chain includes:

- i- A file gamma that combines the camera gamma and the software-encoding gamma ( $\gamma_{\text{file}} = \gamma_{\text{camera}} * \gamma_{\text{encoding}}$ ). In this document we will consider that the camera gamma and the encoding gamma are defined by the same equation, that only one of them is used at one time, and that they simply distinguish the origin of the data.
- ii- A decoding gamma, which is defined as the gamma of any transformation performed by the software reading the image file. In this document we will assume that the software does not modify the gamma once the original file is created and that the decoding gamma is equal to one.
- iii- A display gamma, which combines the lookup-table (LUT) gamma and the CRT gamma: ( $\gamma_{\text{display}} = \gamma_{\text{LUT}} * \gamma_{\text{CRT}}$ ) or ( $\gamma_{\text{display}} = \gamma_{\text{LUT}} / \gamma_{\text{CRT}}$ ) depending on how  $\gamma_{\text{CRT}}$  is defined.
- iv- The overall gamma that combines all the preceding gammas.
- v- The human eye gamma.

The effect of camera gamma is often defined in the form:

$$\begin{aligned}
 V &= (1 + \textit{offset})L^\gamma - \textit{offset} && \text{for } 1 \geq L \geq \textit{transition} \\
 V &= \textit{slope} \times L && \text{for } \textit{transition} > L \geq 0
 \end{aligned}
 \tag{9}$$

where L is the image luminance ( $0 \leq L \leq 1$ ) and V is the corresponding electrical signal (in Volt). An example of the values found for the *offset*, *gamma*, *transition* and *slope* parameters in ITU-R BT.709-3 (a standard for High-Definition TV, HDTV) are:

$$\textit{offset} = 0,099 \qquad \gamma = 0,45 \qquad \textit{transition} = 0,018 \qquad \textit{slope} = 4,5$$

The function is defined by two segments: a linear segment at low light levels, below the defined transition level, which makes the transform less susceptible to noise around zero luminance, and a power segment with a 0,45 exponent. The effect of that exponent is to compress the luminance signal by assigning a larger signal range to dim colors, where the eye is most sensitive, and a small signal range to bright colors.

The offset term of Equation (9) is related to what is generally identified in TVs and monitors as the *black level*, *intensity* or *brightness* control knob. The combination of  $(1 + \textit{offset})$  is related to the *picture*, *gain* or *contrast* knob. It may sound surprising that brightness be associated with a DC level and contrast to a term which controls the maximum luminance level, but these terms were defined in relation to what is perceived, not the mathematical expression. In effect, the eye perceives as a brightness increase a change in the black level more than it does of a change in the gain. Note: in some displays, the brightness and contrast knobs are effectively labeled the reverse of what is “generally” found!

Equation (9) can be approximated by a simpler function of the form:

$$V = L^\gamma \qquad \text{for } 0 \leq L \leq 1 \qquad , \tag{10}$$

with a gamma optimized to fit the data of the detailed transform. Taking ITU-R BT.709-3 again as an example, a best-fit curve can be obtained with the simpler form of Equation (10) and a gamma of 0,519. The two curves are shown superimposed in Figure 4. The simpler form is often retained to improve computing efficiency.

If the image was computer generated, it is customary to apply a simple gamma correction of the form described in Equation (10) with an exponent value that is different between computing platforms. As shown in Table 5, this exponent is usually 0,45 (1/2,2) for sRGB – a space used only in Windows based computers as this text is written. It is 0,56 (1/1,8) for Macintosh and 0,68 (1/1,47) for SGI – formerly called Silicon Graphics Inc. The luminance “L” in Equation (10) corresponds, and is linearly proportional, to either one of the R, G or B channels. The voltage “V” corresponds to the “gamma corrected” coordinates R’, G’, or B’, the values shown in graphic software dialog boxes, even though we seldom, if ever, see the primes against the RGB letters.

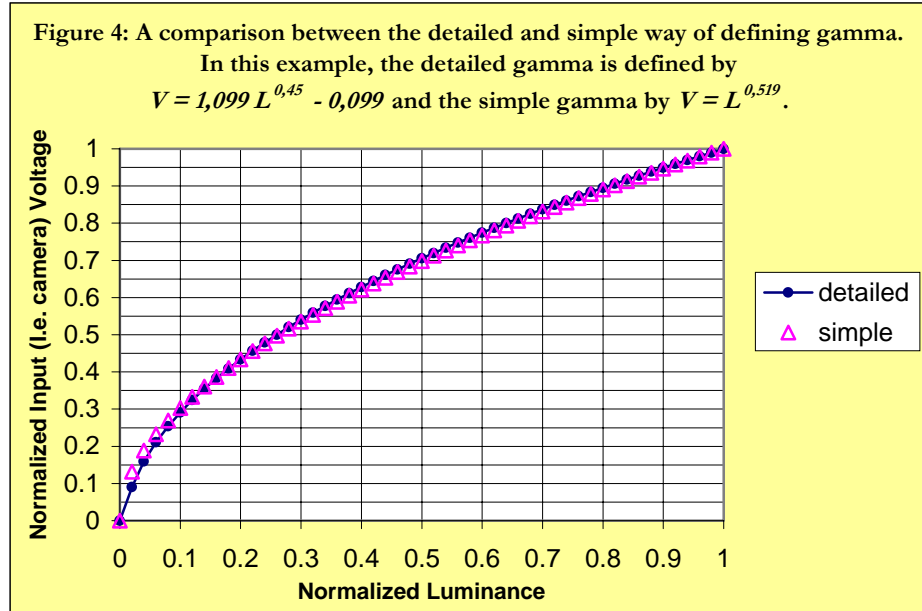
In Windows type PCs, the graphics card LUT is nominally a straight-line one-to-one transfer function. In Apple Macintosh and SGI machines, the graphics card LUT has a transfer function as per Equation (10) with the exponent being 0,69 (1/1,45) for Macintosh and 0,59 (1/1,7) for SGI. It just so happens, and it should not be surprising, that the value of  $(\gamma_{\text{file}} * \gamma_{\text{LUT}})$  is very similar for all platforms.

In many TV standards, a *reference reproducer*, which corresponds to an idealized display, is expressed in a form which is the reverse of the camera transfer function shown as Equation (9):

$$L = \left( \frac{V + \textit{offset}}{1 + \textit{offset}} \right)^{1/\gamma} \tag{11}$$

There again, a simpler, approximate, transfer function can be written:

$$L = V^{1/\gamma} \tag{12}$$



In practice, however, the camera and display gammas are different so that the displayed contrast is higher than the original image contrast. This is done because in dim or dark ambient conditions, a frequent condition for TV viewing, dark tones are perceived brighter than they should and the black to white contrast is lower. Assuming that  $\gamma_{\text{encoding}}$  and  $\gamma_{\text{LUT}}$  are equal to one, a normal assumption for TV work, the ratio between the camera and CRT gammas is typically fixed to 1,25 for dim viewing conditions.

In a properly set monitor for color related work, it is recommended to adjust the black level, or offset, near zero – i.e. barely perceptible from a no-signal state. Also, it is recommended to adjust the video gain – contrast – to maximum value. This is the method used in the Adobe Gamma “Control Panel” tool provided with many Adobe products, and a recent paper by J. R. Jiménez & al.<sup>20</sup> confirms it maximizes the color gamut.

Berns & al.<sup>21</sup> present results of measurements taken on properly set monitors that are best fitted, using Equation (11), with a gamma of 0,406 (1/2,46) and an offset of zero. In this case Equation (11) corresponds exactly to Equation (12). A rounded value of 0,4 (1/2,5) is used as a generic CRT gamma for most spaces in Table 5.

The overall system gamma is:

$$\gamma_{\text{overall}} = \frac{\gamma_{\text{file}} \times \gamma_{\text{LUT}}}{\gamma_{\text{CRT}}} \quad (13)$$

It can be seen in Table 5 that the overall gamma varies between 0,96 and 1,30, with the lower range values associated with color spaces dedicated for computer work. This result is consistent with the brighter illumination conditions typical of computer work and the correspondingly higher, in fact more normally, perceived contrast. At some point however, veiling glare could lower the contrast again. This explains why professional systems have glare protecting hoods around monitors, as well as neutral gray bezels – and sometimes an entirely gray workplace – to prevent unwanted color contamination.

The human eye has a response similar to the one assigned for cameras. The perceived luminance  $L^*$ , called lightness, as described by Equation (3) is essentially the same as Equation (9) but with a 0,33 (1/3) exponent. The camera signals, or encoded file data if the image is generated directly in software, are thus compressed in an efficient way with more signal range associated with the lower brightness colors where the eye has more discrimination. To be viewed, the image goes through the graphics LUT and the CRT electronics, a path that effectively decompresses the recorded signal so that the eye can perceive it as if he saw the original scene, with a more or less serious correction added to account for viewing conditions.

## 2.2 The various RGB color spaces

Defining a color space is a compromise between the availability of good primaries, the signal noise, and the number of digital levels supported by the file type. There is no point in defining a very large gamut if the number of possible colors is so small that the eye will see discrete steps – *banding* – where uniform gradients are required, a typical problem of digital systems, or if the color space gamut is much bigger than the gamut of all output devices. Assigning more bits to each primary is a solution which is now seen more often; it requires more computing power but it minimizes banding due to repetitive image correction and manipulation even if the final image is down-sampled to be compatible with the range of the output device.

Because there is still a large quantity of software which is not able to cope with image files embedded with color calibration profiles, either for competitive or practical reasons, the spaces associated to computing platforms are usually defined relative to a specific reference display. For example, the Apple RGB space is defined with Sony Trinitron phosphors, even though other Apple products, like the iMac, use a CRT from another manufacturer with different characteristics. For all files generated from an Apple platform which do not contain embedded profiles, and which are of unknown origin, the “standard” Apple RGB space should be assumed. In most modern operating systems (Mac OS 9, Windows 98), display calibration is handled by the operating system.

RGB spaces have evolved, sometimes for technological reasons (NTSC to SMPTE-C), sometimes to fulfill professional requirements (ColorMatch, Adobe RGB), and sometimes because that’s how the display was built and it became a, de-facto, standard (Apple RGB). A short description of many RGB spaces follows; detailed specifications are shown in Table 5. The gamut of many of them can be visualized in Figures 3a and 3b.

### Adobe RGB

Formerly known as SMPTE-240M for Photoshop user, this space has been renamed once the final SMPTE-240M standard committee settled for a smaller gamut. Adobe RGB is very close to the original NTSC space and represents a good compromise between gamut size and the number of colors available in an 8 bit per primary system. However, if available, 16 bit per primary should be preferred. While a relatively large number of colors cannot be printed using the SWOP process, particularly in the green portion of the gamut, newer printing processes, such as Pantone Hexachrome, take advantage of this space.

### Apple RGB

A very common RGB space on the desktop that is similar in gamut size to the ColorMatch and sRGB spaces. The Apple RGB, like the ColorMatch and SGI spaces, has a non-unity display LUT gamma which is compensated by the file encoding gamma. Apple specifies its displays in terms of display gamma as per the definition mentioned in Section 2.1.6. When a value of 1,8 is entered by the user in the control panel for display gamma, the LUT is filled with numbers corresponding to a gamma equal to  $1,8/2,6=0,69$  (or 1,45 if you define gamma using the reciprocal value).

ColorSync, Apple’s color management technology at the operating system level, automatically takes care of color calibration for all input and output devices and can be used to convert files from one space to another. For example, the first iMac display, manufactured by LG Electronics, had the following phosphor chromaticities, different from the Apple RGB space values. Its gamma was specified as 1,8, or 0,56 ( $= 1/1,8$ ), and an adjustable white point.

iMac phosphors			
	R	G	B
x	0,610	0,298	0,151
y	0,342	0,588	0,064

### CIE RGB

A relatively large gamut space specified by the monochromatic primaries at 435,8, 546,1, and 700 nm that were used in the experiments at the origin of the color-matching functions. The gamma values shown for this space are “generic”; for instance, an encoding gamma of 0,455 ( $1/2,2$ ) is assigned by default to this space by Adobe Photoshop.

### ColorMatch RGB

This space was originally devised by Radius to be used in conjunction with its PressView line of calibrated displays dedicated to professional use. Often favored over other desktop spaces by critics, the gamut of this space is not significantly larger than the Apple RGB or sRGB. For example, compared with sRGB, it has a slightly larger gamut in the blue-green region but a smaller one in the red-blue region.

RGB space	Primaries / Phosphors			White Illuminant	XYZ to RGB matrix	RGB to XYZ matrix	Power Functions Exponents, i.e. gamma ( $\gamma$ )					
	R	G	B				encoding gamma "detailed"	$\gamma$ for each element of the imaging chain				
<b>Adobe (1998)</b>	Adobe RGB (1998)			D65	<b>XYZ to RGB (Adobe)</b>		<b>RGB (Adobe) to XYZ</b>		N.A.	"simple" encoding:	0,45 (2,20)	
	x :	0,6400	0,2100	0,1500	0,3127	2,0414	-0,5649	-0,3447		LUT:	1	
	y :	0,3300	0,7100	0,0600	0,3290	-0,9693	1,8760	0,0416		CRT:	0,40 (2,50)	
	z :	0,0300	0,0800	0,7900	0,3583	0,0134	-0,1184	1,0154		overall:	1,14	
<b>Apple</b>	Trinitron			D65	<b>XYZ to RGB (Apple)</b>		<b>RGB (Apple) to XYZ</b>		N.A.	"simple" encoding:	0,56 (1,80)	
	x :	0,6250	0,2800	0,1550	0,3127	2,9516	-1,2894	-0,4738		LUT:	0,69 (1,45)	
	y :	0,3400	0,5950	0,0700	0,3290	-1,0851	1,9909	0,0372		CRT:	0,40 (2,50)	
	z :	0,0350	0,1250	0,7750	0,3583	0,0855	-0,2695	1,0913		overall:	0,96	
<b>CIE</b>	CIE RGB			E	<b>XYZ to RGB (CIE)</b>		<b>RGB (CIE) to XYZ</b>		N.A.	"simple" encoding:	0,45 (2,20)	
	x :	0,7350	0,2740	0,1670	0,3333	2,3707	-0,9001	-0,4706		LUT:	1	
	y :	0,2650	0,7170	0,0090	0,3333	-0,5139	1,4253	0,0886		CRT:	0,40 (2,50)	
	z :	0,0000	0,0090	0,8240	0,3333	0,0053	-0,0147	1,0094		overall:	1,14	
<b>ColorMatch</b>	P22-EBU			D50	<b>XYZ to RGB (P22-EBU)</b>		<b>RGB (P22-EBU) to XYZ</b>		N.A.	"simple" encoding:	0,56 (1,80)	
	x :	0,6300	0,2950	0,1500	0,3457	2,6423	-1,2234	-0,3930		LUT	0,56 (1,80)	
	y :	0,3400	0,6050	0,0750	0,3585	-1,1120	2,0590	0,0160		and CRT:	(combined)	
	z :	0,0300	0,1000	0,7750	0,2958	0,0822	-0,2807	1,4560		overall:	1,00	
<b>HDTV (HD-CIF)</b>	HDTV (ITU-R BT.709-5)			D65	<b>XYZ to RGB (R709)</b>		<b>RGB (R709) to XYZ</b>		offset:	0,099	"simple" encoding:	<b>0,51</b> (1,95)
	x :	0,6400	0,3000	0,1500	0,3127	3,2405	-1,5371	-0,4985	$\gamma$ :	<b>0,45</b>	LUT:	1
	y :	0,3300	0,6000	0,0600	0,3290	-0,9693	1,8760	0,0416	transition:	0,018	CRT:	0,40 (2,50)
	z :	0,0300	0,1000	0,7900	0,3583	0,0556	-0,2040	1,0572	slope:	4,5	overall:	1,28
<b>NTSC (1953)</b>	NTSC (1953)			C	<b>XYZ to RGB (NTSC)</b>		<b>RGB (NTSC) to XYZ</b>		offset:	0,099	"simple" encoding:	<b>0,51</b> (1,95)
	x :	0,6700	0,2100	0,1400	0,3101	1,9100	-0,5325	-0,2882	$\gamma$ :	<b>0,45</b>	LUT:	1
	y :	0,3300	0,7100	0,0800	0,3161	-0,9847	1,9992	-0,0283	transition:	0,018	CRT:	0,40 (2,50)
	z :	0,0000	0,0800	0,7800	0,3738	0,0583	-0,1184	0,8976	slope:	4,5	overall:	1,28
<b>PAL / SECAM</b>	EBU 3213 / ITU			D65	<b>XYZ to RGB (EBU)</b>		<b>RGB (EBU) to XYZ</b>		offset:	0,099	"simple" encoding:	<b>0,51</b> (1,95)
	x :	0,6400	0,2900	0,1500	0,3127	3,0629	-1,3932	-0,4758	$\gamma$ :	<b>0,45</b>	LUT:	1
	y :	0,3300	0,6000	0,0600	0,3290	-0,9693	1,8760	0,0416	transition:	0,018	CRT:	0,40 (2,50)
	z :	0,0300	0,1100	0,7900	0,3583	0,0679	-0,2289	1,0694	slope:	4,5	overall:	1,28
<b>SGI</b>	Trinitron			D65	<b>XYZ to RGB (SGI)</b>		<b>RGB (SGI) to XYZ</b>		N.A.	"simple" encoding:	0,68 (1,47)	
	x :	0,6250	0,2800	0,1550	0,3127	2,9516	-1,2894	-0,4738		LUT:	0,59 (1,70)	
	y :	0,3400	0,5950	0,0700	0,3290	-1,0851	1,9909	0,0372		CRT:	0,35 (2,86)	
	z :	0,0350	0,1250	0,7750	0,3583	0,0855	-0,2695	1,0913		overall:	1,14	
<b>SMPTE-240M</b>	SMPTE-C			D65	<b>XYZ to RGB (240M)</b>		<b>RGB (240M) to XYZ</b>		offset:	0,112	"simple" encoding:	<b>0,52</b> (1,92)
	x :	0,6300	0,3100	0,1550	0,3127	3,5054	-1,7395	-0,5440	$\gamma$ :	<b>0,45</b>	LUT:	1
	y :	0,3400	0,5950	0,0700	0,3290	-1,0691	1,9778	0,0352	transition:	0,023	CRT:	0,40 (2,50)
	z :	0,0300	0,0950	0,7750	0,3583	0,0563	-0,1970	1,0502	slope:	4,0	overall:	1,30
<b>SMPTE-C</b>	SMPTE-C			D65	<b>XYZ to RGB (SMPTE-C)</b>		<b>RGB (SMPTE-C) to XYZ</b>		offset:	0,099	"simple" encoding:	<b>0,51</b> (1,95)
	x :	0,6300	0,3100	0,1550	0,3127	3,5054	-1,7395	-0,5440	$\gamma$ :	<b>0,45</b>	LUT:	1
	y :	0,3400	0,5950	0,0700	0,3290	-1,0691	1,9778	0,0352	transition:	0,018	CRT:	0,40 (2,50)
	z :	0,0300	0,0950	0,7750	0,3583	0,0563	-0,1970	1,0502	slope:	4,5	overall:	1,28
<b>sRGB</b>	HDTV (ITU-R BT.709-5)			D65	<b>XYZ to RGB (R709)</b>		<b>RGB (R709) to XYZ</b>		offset:	0,055	"simple" encoding:	<b>0,45</b> (2,20)
	x :	0,6400	0,3000	0,1500	0,3127	3,2405	-1,5371	-0,4985	$\gamma$ :	<b>0,42</b>	LUT:	1
	y :	0,3300	0,6000	0,0600	0,3290	-0,9693	1,8760	0,0416	transition:	0,003	CRT:	0,40 (2,50)
	z :	0,0300	0,1000	0,7900	0,3583	0,0556	-0,2040	1,0572	slope:	12,92	overall:	1,14
<b>Wide Gamut</b>	700 / 525 / 450 nm			D50	<b>XYZ to RGB (Wide)</b>		<b>RGB (Wide) to XYZ</b>		N.A.	"simple" encoding:	0,45 (2,20)	
	x :	0,7347	0,1152	0,1566	0,3457	1,4625	-0,1845	-0,2734		LUT:	1	
	y :	0,2653	0,8264	0,0177	0,3585	-0,5228	1,4479	0,0681		CRT:	0,40 (2,50)	
	z :	0,0000	0,0584	0,8257	0,2958	0,0346	-0,0958	1,2875		overall:	1,14	

Table 5: Colorimetric specifications of various RGB spaces and transform matrices between RGB space and CIE 1931 XYZ values.

The main advantages for its users are a reproducible and well-characterized environment. A calibrated PressView system takes into account, independently for each RGB channel, the CRT gain, offset and brightness combined with the display LUT, which it uses for calibration purposes. The resulting system, between the graphics file and the eye, has a “perfect” 0,56 (1/1,8) gamma on a 0,33 cd/m<sup>2</sup> black pedestal and a white point luminance of 85 cd/m<sup>2</sup>.

The primaries shown in Table 5 are different than the ones used in Photoshop. The ones in this document were confirmed by miro displays,<sup>22</sup> which recently purchased the Radius brands and technologies from Radius, now renamed Digital Origin.

### **HDTV RGB and sRGB**

Identical in terms of gamut, these two spaces differ only in their definition of the viewing conditions, which are simply assumed in ITU-R BT.709-3, a High-Definition-TV (HDTV) standard, and precisely defined in IEC 61966-2-1, the sRGB standard (see Table 6 for a more complete description of these standards). With chromaticities not very far from SMPTE-C (and SMPTE-240M), they strive to represent the evolution of our standard TV and its convergence with the PC world, while maintaining compatibility with the large quantity of recorded media.

Advertised as a general-purpose space for consumer use, sRGB is proposed for applications where embedding the space profile (ex: ICC profile) may not be convenient for file size or compatibility purposes. By having all elements in a system sRGB compliant, no time is lost in conversions. The World Wide Web is obviously a target of choice for this space but it should not be discounted for other “scanner-to-printer” applications. An extended gamut color encoding standard has been proposed for sRGB; it supports multiple levels of precision while being compatible with the base standard.

### **NTSC RGB**

The color space of the first North-American TV sets. It is now an obsolete space that has been replaced by one defined with more efficient – brighter – phosphors, SMPTE-C, albeit at the expense of the gamut size. In a strange turn of events, the Adobe RGB space, which was devised mostly for printed graphics applications, is very similar to this space, a sign of the significant recent progress in the printing industry.

### **PAL / SECAM RGB**

The current 50 Hz television standard. Very similar in gamut size to the current North-American standard (SMPTE-C), and to Apple RGB, SGI RGB and sRGB.

### **SGI RGB**

The chromaticities of a Sony Trinitron CRT are shown but other displays by Hitachi and Mitsubishi, with different chromaticities, are also found in the SGI product line. The relatively low gamma of 0,35 (or high gamma, if you consider the reverse value of 2,86 quoted as being the gamma by the tube’s manufacturer) is common for Sony’s GDM series of displays from which the SGI-Sony displays are derived.

When a gamma number is entered by the user in an SGI system, the LUT is filled with values corresponding to a  $\gamma_{LUT} = 1/\text{gamma\_number}$ .

### **SMPTE-C and SMPTE-240M RGB**

SMPTE-C defines the primaries for the current North American and Japanese standard, SMPTE 170-1999. You should note that, for compatibility with existing studio equipment, the primaries of NTSC are also accepted in SMPTE 170-1999. The gamma defined in SMPTE 170-1999 is identical to the one defined in ITU-R BT709.3, which is slightly different than the one defined in SMPTE-240M. However, a simple gamma of 0,455 (1/2,2) is often used in computer software for both spaces. The single parameter gamma values in Table 5 (“simple gamma”) were obtained by deriving a best fit on the detailed gamma function.

### **Wide gamut RGB**

As its name says, this is an extremely wide gamut. It is based on monochromatic primaries at 700, 525, and 450 nm. Although possible to generate these wavelengths with lasers, the red primary is in a region where the response of the eye is quite low. Since a 700 nm red requires significantly more power to match the brightness of the other two wavelengths, and since cost is proportional to power, it is more cost-effective to use a shorter red wavelength. As mentioned previously, proposed wavelengths for optimum luminous efficiency for a display with three primaries are 610, 530, and 450 nm.

### 3 Television and multimedia systems

Recent years have seen an explosion of new standards following the development of analog HDTV, which has really only caught on in Japan, and now digital TV (DTV) and HDTV. We also have witnessed the emergence of multimedia, a new form of information distribution mixing text, images, video, sound and interaction, which was rendered possible with the development of powerful personal computers. The commonalities of many parameters of the various RGB spaces defined for the television and multimedia fields show how intertwined these technologies have become. Eventually, some may hope that they are the same, but history has shown that new requirements will most likely create a need for new standards. For example, we could well standardize now on ITU-R BT.709-3 and the derivative IEC 61966-2-1 (sRGB) for all TV and computer work. However, this solution would not meet the requirements of the printing industry that, in many instances, needs displays with a larger color gamut than what can be obtained with current CRT phosphors. Also, HDTV itself is in constant evolution and the operating infrastructure, which requires new equipment for production and transmission, is far from being ready. In such an environment, the best bet is to keep abreast of as many standards as we can, if only because a lot of content will be created with these interim systems which will then have to be translated to the final colorimetric characteristics.

The colorimetric and opto-electronic transfer characteristics of major TV standards as well as a short content description are presented in Table 6. Most of these standards can now be purchased “online” from the organizations Web sites.<sup>23</sup> The rightmost column, “opto-electronic transfer characteristic”, is the definition of the input, or camera, gamma with parameters to be used as per Equation (9). The corresponding reference reproducer is not always specified, but when it is, it is the reverse equation from these parameters. When the transfer characteristic is identical to another standard, the number of the reference standard, or the standard often quoted as such, is shown in place of the characteristics. Not all standards characterize opto-electronic transfer and the mention “N.A.” (Not Applicable) is shown in these cases.

Phosphors primaries and white point are shown, when applicable. Some standards propose different primaries-white point combinations for specific cases. The number of the standard, or its well-recognized industry name, such as SMPTE-C, is shown when primaries are identical to the reference standard.

The display ratio, the ratio of the horizontal image size on the vertical image size, is either 4:3 or 16:9. The squarer 4:3 ratio is the one used in our current TV systems (NTSC, PAL, SECAM). The “letterbox” 16:9 ratio is used for the many flavors of HDTV.

A TV image, or frame, is composed of lines that have traditionally been displayed in two sequences, or fields. The two fields are *interlaced*; the first field contains the odd lines and the second field the even lines. The field rate is twice the frame rate. This scheme was done to maximize the apparent resolution while minimizing the jitter that would be noticeable if the entire image was updated at the lower frame rate. This interlace is noted as 2:1.

With the improvement in electronic devices performance, it is now possible, if not always cost-effective, to send all the frame in one field, called *progressive scan*, and noted as 1:1. All computer displays as well as many of the various HDTV modes work on this principle. However, due to equipment and bandwidth related cost reasons, a 2:1 interlace will still be used in HDTV.

The frame rates have historically been close or equal to the power line frequencies, 59,94 Hz in North America and 50 Hz in Europe. Compatibility with the vast number of sets and already recorded media imposes a continued support for these frequencies.

There is a tradition of specifying TV displays in terms of “lines” for both the vertical and horizontal resolution. The vertical resolution is fixed by design but the horizontal resolution, coming from an analog signal, is dependent of the signal bandwidth. A top-of-the-line higher-bandwidth TV will be specified as capable of showing more horizontal lines than a lower cost model. Such is not the case in computers and in digital TVs where the number of pixels is fixed both horizontally and vertically, with the caveat that the actual resolution always depends on the resolution of the source image and the display physical dot pitch.

Also, the number of visible, or active, vertical lines is always smaller than the total number of lines in a frame. The hidden lines are used for screen refresh, to enable the electron beam in a CRT to get back to the top of the display, as well as to transmit non-image information, such as captions.

standard	title	additional information	primaries (white)	opto-electronic transfer characteristic	luma (E'Y)
RP 145-1994 (SMPTE)	SMPTE C Color Monitor Colorimetry	Current North American and Japanese broadcast standard. No aspect and interlace ratios, sampling, resolution, number of lines, and picture rate specification in this standard.	R G B x: 0,6300 0,3100 0,1550 y: 0,3400 0,5950 0,0700 (D65)	N.A.	N.A.
SMPTE 170M-1999	Television - Composite Analog Video Signal - NTSC for Studio Applications	Current North American and Japanese broadcast standard. 4:3 ratio. 525/59,94/2:1.	SMPTE C (i.e.: RP-145) (D65) and NTSC (1953): R G B x: 0,6700 0,2100 0,1400 y: 0,3300 0,7100 0,0800	Same as ITU-R BT.709-3	ITU-R BT.601-5
ANSI/SMPTE 240M-1995	Television - Signal Parameters - 1125-Line High Definition Production Systems	16:9 ratio, 1035 active lines per frame. 1125/60/2:1 and 1125/59,94/2:1.	SMPTE C (D65)	offset: 0,115 $\gamma$ : 0,45 transition: 0,0228 slope: 4,0	0,212 E'R + 0,701 E'G + 0,087 E'B
SMPTE 260M-1999	Television - 1125/60 High-Definition Production System - Digital Representation and Bit-Parallel Interface	Digital representation of the 1125/60 ANSI/SMPTE 240M signal parameters, plus the mechanical and electrical interface. 16:9 ratio.	SMPTE C (D65)	ANSI/SMPTE 240M	ANSI/SMPTE 240M
SMPTE 274M-1998	Television - 1920 x 1080 Scanning and Analog and Parallel Digital Interfaces for Multiple Picture Rates	16:9 ratio, 1125 total lines per frame. 1920 x 1080/60, /59,94, and /50 in both 1:1 and 2:1 interlace. 1920 x 1080/30, /29,97, /25, /24, and /23,98 in 1:1 only.	ITU-R BT.709-3 (D65)	ITU-R BT.709-3	ITU-R BT.709-3
ANSI/SMPTE 293M-1996	Television - 720 x 483 Active Line at 59,94-Hz Progressive Scan Production - Digital Representation	Principally defined for the production of content for EDTV-II (NTSC letterbox compatible with SMPTE 170M).	SMPTE C (D65)	ITU-R BT.709-3	ITU-R BT.601-5
ANSI/SMPTE 295M-1997	Television - 1920 x 1080 50-Hz - Scanning and Interface	Analog and digital, 16:9 ratio, 1250 total lines per frame. 1920 x 1080/50 in both 1:1 and 2:1 interlace.	ITU-R BT.709-3 (D65)	ITU-R BT.709-3	ITU-R BT.709-3
ANSI/SMPTE 296M-1997	Television - 1280 x 720 Scanning, Analog and Digital Representation and Analog Interface	Analog and digital, 16:9 ratio, 750 total lines per frame. 1280 x 720/60/1:1 and 1280 x 720/59,94/1:1.	ITU-R BT.709-3 (D65)	ITU-R BT.709-3	ITU-R BT.709-3
EBU Tech. 3213 and other documents.	EBU standard for chromaticity tolerances for studio monitors (1975, re-issued 1981, Out-of-Print)	Current European broadcast standard. PAL / SECAM. 625/50.	R G B x: 0,6400 0,2900 0,1500 y: 0,3300 0,6000 0,0600	Assumed gamma of receiver: 2,8	ITU-R BT.601-5
IEC 61966-2-1 (final draft)	Multimedia systems and equipment - Colour measurement and management - Part 2-1: Colour management - Default RGB colour space - sRGB	No aspect and interlace ratios, sampling, resolution, number of lines, and picture rate specification in this standard.	ITU-R BT.709-3 (D65)	offset: 0,055 $\gamma$ : 1 / 2,4 transition: 0,0031308 slope: 12,92	N.A.
ITU-R BT.470-6	Conventional television systems	Worldwide inventory of specifications for recommended NTSC, PAL, and SECAM systems and variants. Analog, 4:3 ratio, 2:1 interlace. 525/60, 525/59,94, and 625/50.	NTSC (1953) and (C): for M/NTSC, M/PAL and some PAL variants (permitted for existing SECAM sets). EBU 3213 and (D65): for most PAL and SECAM variants.	Assumed gamma of receiver: 2,2 for M/NTSC and some M/PAL variants; 2,8 for M/PAL and others.	ITU-R BT.601-5
ITU-R BT.601-5	Studio encoding parameters of digital television for standard 4:3 and wide-screen 16:9 aspect ratios	4:3 and 16:9 ratios, 13,5 MHz: 4:2:2, 525/60 and 625/50; 4:4:4, 525/60 and 625/50. 16:9 ratio, 18 MHz: 4:2:2, 525/60 and 625/50; 4:4:4, 525/60 and 625/50.	N.A.	N.A.	0,299 E'R + 0,587 E'G + 0,114 E'B
ITU-R BT.709-3	Parameter values for the HDTV standards for production and international programme exchange	Analog and digital, 16:9 ratio. ITU-R BT.709-3 luma used for systems related to conventional TV: 1125/60/2:1, and for square pixel common image format (HD-CIF): 1125/60 (comprises 1080/60/2:1, 1080/59,94/2:1, 1080/60/1:1, and 1080/59,94/1:1), and 1250/50 (comprises 1080/50/2:1 and 1080/50/1:1). ITU-R BT.601-5 luma used only for systems related to conventional TV: 1250/50/2:1 (1152 active lines).	R G B x: 0,6400 0,3000 0,1500 y: 0,3300 0,6000 0,0600 (D65)	offset: 0,099 $\gamma$ : 0,45 transition: 0,018 slope: 4,5	0,2126 E'R + 0,7152 E'G + 0,0722 E'B  ITU-R BT.601-5 also specified, see additional information.
ITU-R BT.1358	Studio parameters of 625 and 525 line progressive scan television systems	Analog and digital, 4:3 and 16:9 ratios. 625/50/1:1, 576 active lines per picture, and 525/59,94/1:1, 483 active lines per picture.	EBU 3213 for 625 line system SMPTE C for 525 line system (D65)	ITU-R BT.709-3	ITU-R BT.601-5

Table 6: Colorimetric and opto-electronic transfer characteristics of major TV standards. These standards are defined by the following organizations: ANSI: American National Standards Institute; EBU: European Broadcasting Union; IEC: International Electrotechnical Commission; ITU: International Telecommunication Union; SMPTE: Society of Motion Picture and Television Engineers. See Reference 23 for Web sites addresses.



There is no published industry-wide standard for the colorimetric characteristics of personal computers and each platform (Apple, SGI, etc.) has its own set of requirements. Moreover, until the recently completed IEC 61966-2-1 standard for sRGB, there was virtually no colorimetric information available for Intel/Windows compatible computers. In particular, for this open platform, there is not much control on the characteristics of the display phosphors, which can vary between manufacturers and between models. For critical work, specific tristimulus coordinates can usually be found in the user manual but the specifications of ITU-R BT.709-3 (sRGB) should be used if the images are going to be distributed and looked at on unknown displays. This is not as bad as it looks since, in modern displays, the chromaticities are quite close, at least between CRTs, and it is possible to adjust the color temperature at specific values.

In terms of resolution, computer displays, with their multi-sync capabilities, have fewer ties with the past in terms of frame rate and resolution. For example, in the IBM compatible world, each step up, from VGA at 640 by 480 pixels to UXGA at 1600 by 1200 pixels, is backward compatible with the data generated with the previous generations.

An overview of the colorimetric characteristics of TVs and displays would not be complete without a description of *Luma*. Luma is an electrical TV signal on which the luminance of the image is encoded. It is the only signal, apart from the synchronizing signals, which is required for black and white TV. The term Luma is preferred to luminance since it is determined from gamma corrected signals, whereas luminance is determined from physical, linear, data. Confusion may still arise however, because Luma coefficients are sometimes identical to the luminance coefficients, the middle row, of the RGB to XYZ matrix.

Luma coefficients are **not** used in any part of the XYZ to RGB conversion; they are simply a way of encoding luminance for recording or transmission purposes. There are basically three Luma definitions used in all TVs worldwide; they are shown in Table 7.

standard	Luma (E'y)
ANSI/SMPTE 240M-1995	0,212 E'r + 0,701 E'g + 0,087 E'b
ITU-R BT.601-5	0,299 E'r + 0,587 E'g + 0,114 E'b
ITU-R BT.709-3	0,2126 E'r + 0,7152 E'g + 0,0722 E'b

Table 7: The principal Luma definitions found in TV standards.

Another important specification of a display system is represented by a series of three numbers such as “4:2:2”. These numbers are the relative bandwidths of the Luma and the two color signals – *chrominance* – into which the image is encoded. In ITU-R BT.601-5, for example, the 4:2:2 specification at 13,5 MHz means that the two chrominance signals are encoded at 6,75 MHz and the Luma signal at 13,5 MHz. The lower bandwidth for chroma is a common display configuration that takes advantage of the eye higher sensitivity to luminance resolution relative to color resolution. In particular, the eye cannot well resolve bluish colors but is quite sensitive to their shades.

In Table 6, the “additional information” column contains information on aspect and interlace ratios, sampling, resolution, number of lines, and picture rate specification. For instance, the description for SMPTE 274M, an analog and digital HDTV standard, as shown in Table 6 is:

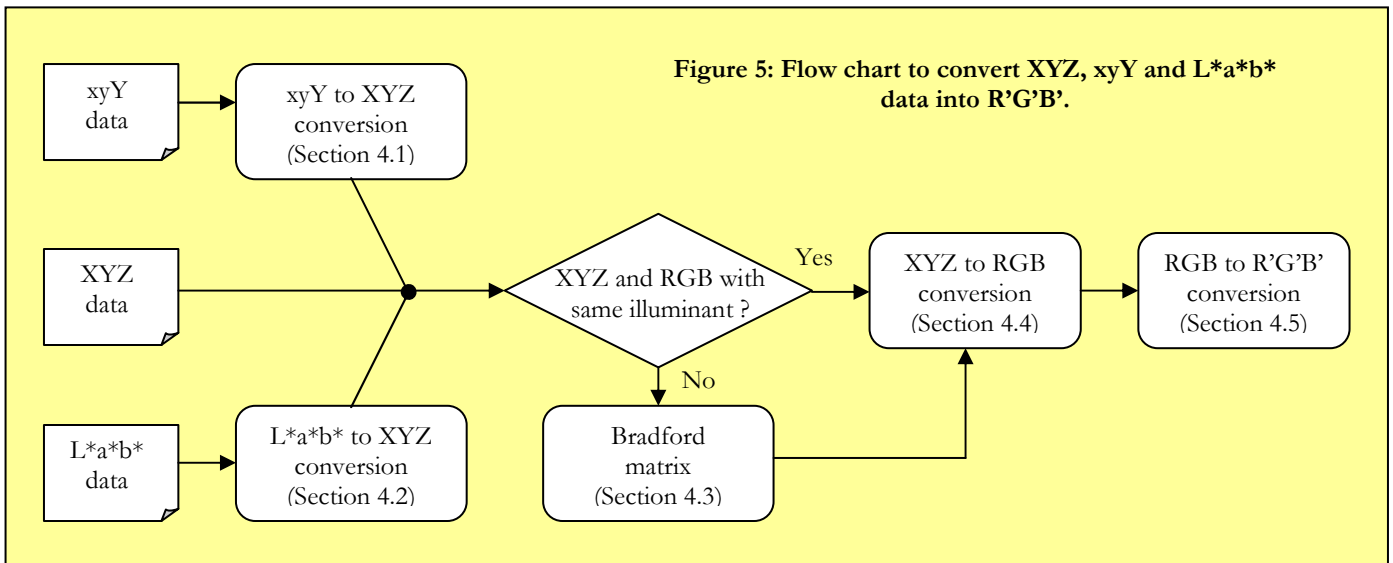
*16:9 ratio, 1125 total lines per frame  
1920 × 1080/60, /59,94, and /50 in both 1:1 and 2:1 interlace  
1920 × 1080/30, /29,97, /25, /24, and /23,98 in 1:1 only*

In condensed form, this information describes a total of eleven display formats at 1920 x 1060 resolution: three with 1:1 interlace at 60 Hz, 59,94 Hz, and 50 Hz frame rates; three with 2:1 interlace at 60 Hz, 59,94 Hz, and 50 Hz frame rates; five with 1:1 interlace at 30 Hz, 29,97 Hz, 25 Hz, 24 Hz and 23,98 Hz frame rates. All formats have a 16:9 image size ratio and a total of 1125 lines.

## 4 From xyY to R'G'B'

Figure 5 shows a flow chart of the steps required to convert xyY data to a specific gamma encoded RGB space. The chart also covers starting with XYZ and L\*a\*b\* data since these representations are often used. The parameters of many common RGB spaces can be found in Table 5 but the procedure provided in this document can be applied to any variant or specific need. The steps are detailed in the following sections.

All these conversions can easily be performed with a spreadsheet. You may also want to make a stand-alone program from scratch from the basic equations. A few software libraries were developed for color conversions. Some are freely available on the Web and often found in universities' sites. Of course, commercial color conversion program do exist but they are usually associated with hardware.



- For data available as xyY coordinates, start with the simple conversion described in Section 4.1.
- If the data is available in XYZ format, and if it was taken with the same illuminant as the target RGB space, you can go directly to Section 4.4.
- If the data is available in XYZ format, but was taken with a different illuminant as the target RGB space, you first have to convert it using the Bradford matrix method described in Section 4.3.
- For data available in L\*a\*b\* format, go to Section 4.2.

### 4.1 From xyY to XYZ

For data available in the xyY format, we simply need to reverse the XYZ to xyY conversion described earlier. From Equations (1) and (2) we obtain:

$$X = x(Y / y) \quad \text{and} \quad (14)$$

$$Z = (1 - x - y)(Y / y) \quad . \quad (15)$$

From here you can go to Section 4.3.

## 4.2 From L\*a\*b\* to XYZ

Transforming L\*a\*b\* data to XYZ requires knowing at least the XYZ coordinates of the illuminant used for measurements (defined as  $X_n$ ,  $Y_n$ , and  $Z_n$ ). Coordinates of standard illuminants can be found in Table 3. For non-standard illuminants, you will need to determine their XYZ coordinates either with a colorimeter or with the method described in Table 1.

The XYZ coordinates are found by inverting the set of L\*a\*b\* relations of Equation (3) presented in Section 2.1.3:

$$\begin{array}{l}
 \text{For } L^* \leq 8,000: \\
 X = X_n \left[ (L^*/903,3)^{1/3} + (a^*/500) \right]^3 \\
 Y = Y_n (L^*/903,3) \\
 Z = Z_n \left[ (L^*/903,3)^{1/3} - (b^*/200) \right]^3
 \end{array}
 \quad
 \left.
 \begin{array}{l}
 \text{For } L^* > 8,000: \\
 X = X_n \left[ ((L^*+16)/116) + (a^*/500) \right]^3 \\
 Y = Y_n \left( (L^*+16)/116 \right)^3 \\
 Z = Z_n \left[ ((L^*+16)/116) - (b^*/200) \right]^3
 \end{array}
 \right\} (16)$$

If your data was taken with the same illuminant as the target RGB space, you can go directly to Section 4.4. If not, you should proceed to the next section.

**Note:** When transforming XYZ data back to L\*a\*b\*, using Equation (3), make sure that the  $X_n$ ,  $Y_n$ , and  $Z_n$  coordinates are the ones of the illuminant used to measure XYZ. If you want L\*a\*b\* coordinates for a different illuminant, you first have to convert the XYZ data to the new illuminant using a Bradford matrix.

## 4.3 From XYZ (Source illuminant) to XYZ (Destination illuminant) – Bradford Matrix

The Bradford matrix transform should be applied if the illuminant used to determine the XYZ coordinates of the original data (the source) is different from the illuminant of the target RGB space (the destination). Bradford matrices for often-required transforms are presented in Table 4. For other sets of standard illuminants, a Bradford matrix can be determined using the method shown in Section 2.1.5 from illuminant data found in Table 3.

For non-standard illuminants, you will have to determine their XYZ coordinates either with a colorimeter or with the method described in Table 1.

The XYZ coordinates corresponding to the illuminant of the target RGB space are:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{dest.} = \begin{bmatrix} \text{Bradford} \\ 3 \times 3 \\ \text{matrix} \end{bmatrix} \bullet \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{source} \quad (17)$$

## 4.4 From XYZ to RGB, and vice-versa

The XYZ to RGB matrices for various spaces are shown in Table 5. The RGB triads are obtained with the following multiplication:

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} XYZ \rightarrow RGB \\ 3 \times 3 \\ \text{matrix} \end{bmatrix} \bullet \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad (18)$$

After this operation, the RGB coordinates of the illuminant are (100, 100, 100). All RGB triads should be rescaled at this point – divided by 100 – and the illuminant coordinates be (1, 1, 1). Results over one or below zero are clipped at one and zero respectively.

For spaces not covered in the table, the following procedure based on a recommended practice from the Society of Motion Picture and Television Engineers<sup>24</sup> can be used. This is done by first determining a RGB to XYZ matrix, and then finding the inverse XYZ to RGB matrix.

The conversion between RGB to XYZ is expressed with a 3x3 matrix that has the form:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} RGB \rightarrow XYZ \\ 3 \times 3 \\ matrix \end{bmatrix} \bullet \begin{bmatrix} R \\ G \\ B \end{bmatrix} \quad (19)$$

What this matrix has to perform is map a space defined by three primaries, expressed in xyz coordinates, into a space defined by relative ratios of Red, Green, and Blue. This is easily derived from the special case where RGB = (1, 1, 1), corresponding to pure white, or more precisely, to the illuminating source characterized with known XYZ coordinates. The problem is further simplified since, for a source, we can normalize Y to one, and we can express X and Z, using Equation (1), as  $x_w/y_w$  and  $z_w/y_w$  respectively. The “w” index is an indication these coordinates correspond to the “white” source.

The RGB to XYZ matrix is defined by the xyz chromaticities of the RGB primaries ( $x_R y_R z_R, x_G y_G z_G, x_B y_B z_B$ ) proportioned, with unknown constants  $C_R, C_G$  and  $C_B$ , to meet the goal of the transform. The equation to solve is

$$\begin{bmatrix} x_w / y_w \\ 1 \\ z_w / y_w \end{bmatrix}_{norm\ XYZ\ white} = \begin{bmatrix} (C_R \times x_R) & (C_G \times x_G) & (C_B \times x_B) \\ (C_R \times y_R) & (C_G \times y_G) & (C_B \times y_B) \\ (C_R \times z_R) & (C_G \times z_G) & (C_B \times z_B) \end{bmatrix}_{RGB \rightarrow XYZ} \bullet \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}_{RGB\ white} \quad (20)$$

that can be expressed as the following three equations:

$$\left. \begin{aligned} x_w / y_w &= (C_R \times x_R) + (C_G \times x_G) + (C_B \times x_B) \\ 1 &= (C_R \times y_R) + (C_G \times y_G) + (C_B \times y_B) \\ z_w / y_w &= (C_R \times z_R) + (C_G \times z_G) + (C_B \times z_B) \end{aligned} \right\} \quad (21)$$

to be solved in the usual manner for three equations and three unknowns.

The XYZ to RGB matrix can then be found by inverting the RGB to XYZ matrix with the standard matrix inversion procedure:

$$\begin{bmatrix} XYZ \rightarrow RGB \\ 3 \times 3 \\ matrix \end{bmatrix} = \begin{bmatrix} RGB \rightarrow XYZ \\ 3 \times 3 \\ matrix \end{bmatrix}^{-1} = \frac{adjoint \begin{bmatrix} RGB \rightarrow XYZ \\ 3 \times 3 \\ matrix \end{bmatrix}}{\begin{matrix} determinant \\ RGB \rightarrow XYZ \\ matrix \end{matrix}} \quad (22)$$

where the adjoint of the matrix to invert is divided by its determinant.

A simple way to verify the calculation is to multiply the two inverse matrices together and check that the result is a unitary diagonal matrix. Also, the coefficients of the middle row of the RGB to XYZ matrix, Equation (20), should add up exactly to one (within significant digits) since the special case with Y = 1 when RGB = (1, 1, 1) was used to deduce the matrix.

Now that we have both the XYZ to RGB and RGB to XYZ matrices, we can use them to transform RGB data from one RGB space to another. If the illuminant is not the same for both spaces, we need to apply a Bradford matrix transform in mid process. The RGB space-to-space conversion procedure is represented by the equation:

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix}_{space2} = \begin{bmatrix} XYZ \rightarrow RGB \\ 3 \times 3 \\ Illuminant 2 \end{bmatrix} \bullet \begin{bmatrix} Bradford \\ XYZ(ILL1) \rightarrow XYZ(ILL2) \\ 3 \times 3 \text{ matrix} \end{bmatrix} \bullet \begin{bmatrix} RGB \rightarrow XYZ \\ 3 \times 3 \\ Illuminant 1 \end{bmatrix} \bullet \begin{bmatrix} R \\ G \\ B \end{bmatrix}_{space1} \quad (23)$$

If the illuminant is the same, the Bradford matrix is simply omitted. It is important to mention that converting from one space to another is frequently performed in conjunction with an additional step, called *gamut mapping*, which is not represented in the preceding equation. Gamut mapping algorithms attempt to minimize the effects of clipping by distorting the values of either or both the clipped and non-clipped colors. Variants of the process, still a subject of active research,<sup>25</sup> have been devised for different requirements such as maintaining saturated colors in business graphics or achieving a balanced “realistic” look in pictures, even if none of the resulting colors are accurate. However, you should realize that many RGB to RGB conversion matrices found in the literature are simply the RGB-to-XYZ and XYZ-to-RGB matrices of Equation (23) combined into one, as per ASTM RP 177-93, with no Bradford matrix or gamut mapping.

#### 4.5 From RGB to R'G'B'

Depending on your choice of a detailed or simple gamma, R', G', and B' are determined with either one of the following equations (for simplicity, only R' is shown; G' and B' are similar):

$$\begin{aligned} R' &= \text{round}\left(255 \times \left((1 + \text{offset})R^\gamma - \text{offset}\right)\right) && \text{for } 1 \geq R \geq \text{transition} \\ R' &= \text{round}\left(255 \times \text{slope} \times R\right) && \text{for } \text{transition} > R \geq 0 \end{aligned} \quad (24)$$

or:

$$R' = \text{round}\left(255 \times R^\gamma\right) \quad \text{for } 0 \leq R \leq 1 \quad (25)$$

These equations are similar to Equations (9) and (10) with terms added to scale and round the values to the nearest integer between zero and 255. This scale corresponds to 8 bits per primary, a 24-bit color system.

#### 4.6 Conversion accuracy vs. requirements

Color differences can be expressed mathematically for any space but they make practical sense only for the more uniform spaces where the resulting numbers can be better associated to what the eye perceives. For the L\*a\*b\* space the color difference equation is:

$$\Delta E^*_{ab} = \left[ (\Delta a^*)^2 + (\Delta b^*)^2 + (k\Delta L^*)^2 \right]^{1/2} \quad (26)$$

where k=1 for samples compared in close proximity (k=0,5 or less for samples compared further away from each other, where the eye is less sensitive to lightness differences). A  $\Delta E^*_{ab}=1$  corresponds to colors which are barely differentiable by 50% of a group of observers; the other 50% would see no difference. Even though Equation (26) is a workhouse of the color industry, its statistical threshold is a cause of concern, and of possible litigation, in many industrial applications where expert observers' judgments are confronted. For this reason, better color difference equations are being sought.<sup>26</sup>

When converting from XYZ to R'G'B', beside the inherent errors coming from the accuracy of the original data, the conversion process can introduce additional errors from the number of decimal places used in the conversion matrices constants, from the approximate form of the Bradford matrix, from the clipping required to limit RGB values between zero and one, from the use of a simple gamma instead of a detailed gamma, and from the rounding of the R'G'B' values. Table 8 shows typical errors associated with each operation.

Processing step	Average $\Delta E^*_{ab}$ error	Standard deviation	Notes
Bradford matrix	1,4	0,9	Measured for a D65 to D50 conversion. From Reference 27.
XYZ to RGB (matrix)	$\approx 0$	$\approx 0$	Negligible error when constants with at least 4 significant decimals are used.
XYZ to RGB (clipping)	variable	variable	See text.
RGB to R'G'B' (simple vs. detailed gamma)	1,3	0,92	When a simple gamma expression is used instead of a detailed one (when available). Measured for sRGB.
RGB to R'G'B' (rounding error)	0,23	0,11	Typical values. Values are slightly higher for larger spaces (Ex.: 0,28 average for Adobe (1998)).

Table 8: Typical errors associated to a XYZ to R'G'B' conversion.  
Errors due to clipping are not considered.

A detailed evaluation of the Bradford matrix accuracy was performed on over 1000 colors from the Pantone color data set covering a very large gamut.<sup>27</sup> A first set of color coordinates was determined from spectral data for a D50 illuminant with a method similar to the one shown in Table 2. These coordinates were then converted to a D65 illuminant using the simplified Bradford matrix. The results were compared to a second set of coordinates obtained from spectral data processed with a D65 illuminant. The average  $\Delta E^*_{ab}$  was 1,4 with a standard deviation of 0,9.

The error associated with the Bradford matrix presented above does not include any effect resulting from the precision of the matrix terms. If constants with at least four significant decimals are used, then virtually no error is induced by the mathematics of the conversion. This is also true for the XYZ to RGB matrix.

Clipping error values are not shown in this table since they are very dependent of the specific target space and the gamut of the original data. Clipping will most often be noticed for single-color large-area zones, an annoying situation if that color is associated with a “brand” product. This is where the use of “spot” colors – additional printing plates for dedicated colored inks other than CMY – is justified in many graphic design applications.

Using a simple gamma expression when a detailed one is available adds a  $\Delta E^*_{ab}$  of 1,3 on average with a standard deviation of 0,92, about the same as for the Bradford matrix.

Rounding the R'G'B' introduces an inevitable error of 0,23, on average, which is not noticeable. However, multiple conversions between different RGB spaces could degrade the color fidelity to a point where it could be noticed.

The errors of Table 8 should not be added since they are statistical in nature. The combined effect of multiple processes can be evaluated by calculating the Root-Sum-Squared (RSS) value:

$$\text{Combined\_error} = \left[ (\text{error\#1})^2 + (\text{error\#2})^2 + \dots + (\text{error\#n})^2 \right]^{1/2} \quad (27)$$

As an example, Table 9 shows the error budget associated with a sRGB to ColorMatch conversion. Of course, if the sRGB R'G'B' values were previously calculated with a simple gamma, we can remove this contribution from Table 9 and we are left with the Bradford matrix and the R'G'B' rounding error.

An average  $\Delta E^*_{ab}$  error of 1,9 can be expected for converting between sRGB and ColorMatch, an acceptable result which does not include the effects of clipping which affects only a portion of the conversions. Table 10 shows some characteristics of the clipping errors found in ColorMatch-to-sRGB and sRGB-to-ColorMatch conversions.

Processing steps	Average $\Delta E^*_{ab}$ error
sRGB: R'G'B' to RGB, simple gamma	1,3
sRGB to XYZ	0
Bradford matrix: XYZ <sub>D65</sub> to XYZ <sub>D50</sub>	1,4
XYZ to ColorMatch RGB (matrix)	0
XYZ to ColorMatch RGB (clipping)	Not included (see text)
ColorMatch: RGB to R'G'B' rounding	0,23
<b>Combined RSS error</b>	<b>1,9</b>

Table 9: The error budget associated with a sRGB to ColorMatch conversion. The RGB to R'G'B' conversion is assumed to be with a simple gamma expression.

	sRGB to ColorMatch	ColorMatch to sRGB
Average $\Delta E^*_{ab}$ due to clipping only	0,95	1,1
Standard deviation for $\Delta E^*_{ab}$	1,8	2,2
Conversions clipped at 0	20,6%	16,8%
Conversions clipped at 1	2,4%	4,6%
Conversions clipped at both 0 and 1	0,66%	0%
Conversions clipped	22,4%	21,4%
Maximum $\Delta E^*_{ab}$ error	13,6	11,6
R'G'B' coordinates for maximum error	sRGB (0, 0, 255)	ColorMatch (0, 255, 0)

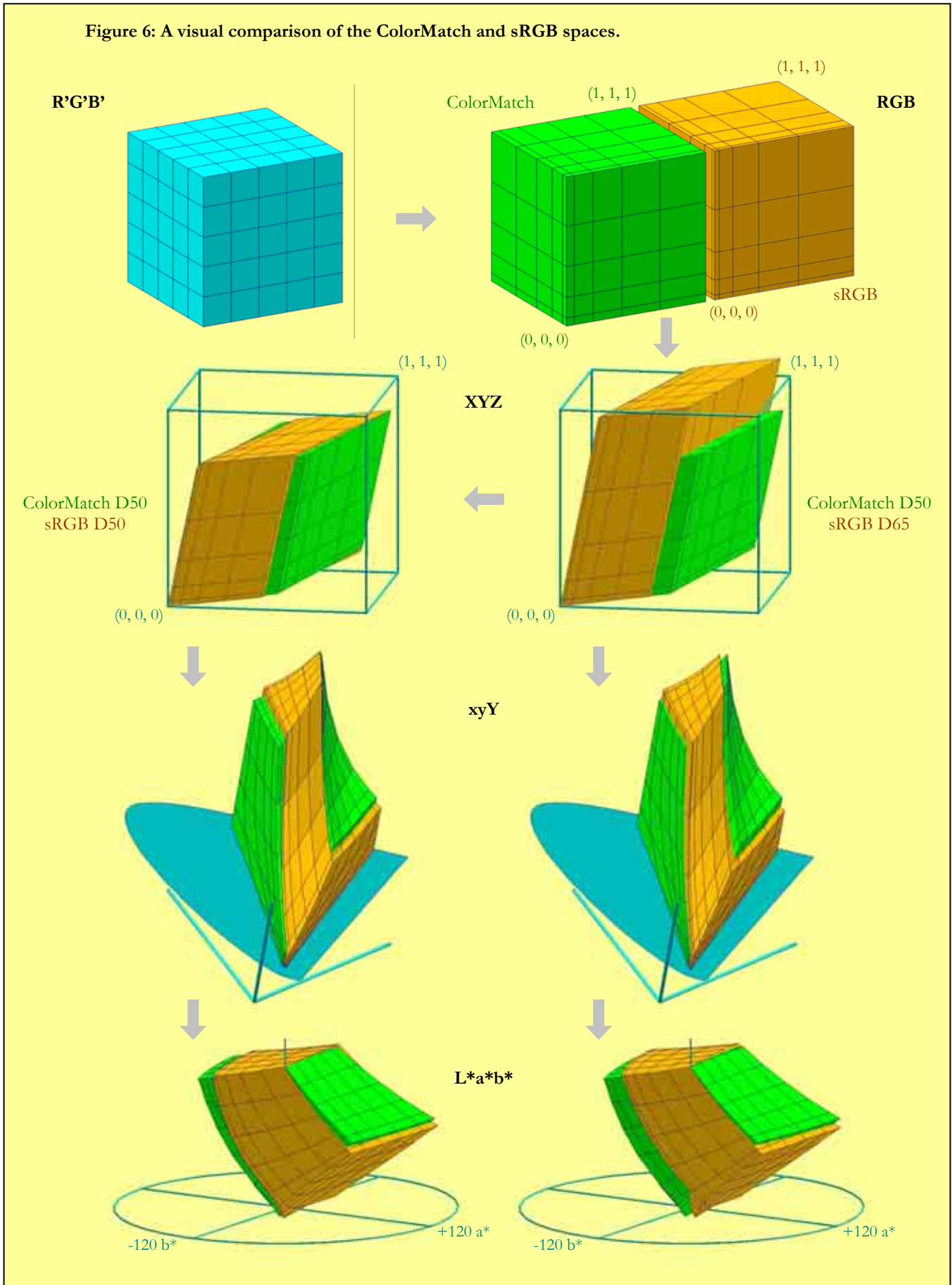
Table 10: Characteristics of the clipping errors found for random samples in ColorMatch-to-sRGB and sRGB-to-ColorMatch conversions. A conversion is considered clipped when one of the R, G, or B values is clipped.

The most surprising result in Table 10 is the high percentage of clipped conversions, a number that cannot be anticipated from a simple comparative observation of the triangular gamut shapes in the chromaticity diagram. Figure 6 shows three-dimensional representations of these two spaces in various coordinate systems. To help visualize how the colors are transformed, the initial R'G'B' cube, identical for both spaces, is divided in 125 uniformly sized smaller cubes. A comparison with the RGB cubes show how the gamma compression assigns more R'G'B' values to lower luminance colors. The RGB cubes are then transformed into XYZ parallelepipeds where the sRGB volume is apparently much larger than the ColorMatch volume. The size difference is no longer obvious when the sRGB XYZ data is transformed from Illuminant D65 to Illuminant D50 using the Bradford matrix. However, even if they are similar and size and almost coincident in space, it just happens that some of the non-coincident zones are for the densely packed low luminance colors. It can easily be seen in the XYZ, xyY and L\*a\*b\* diagrams that almost a complete slice of the shapes, corresponding to 20% of the gamut, is not comprised in the other space. As for which gamut of the two is larger, the answer is a tie since their L\*a\*b\* volumes – used as simplistic estimates for this hard to answer question – are identical within a fraction of a percent.

The largest clipping error occurs for pure blue in the sRGB to ColorMatch conversion, and for pure green in the ColorMatch to sRGB conversion. However, even if the maximum error due to clipping is higher in the sRGB to ColorMatch conversion, there are a higher number of conversions with larger errors when going from ColorMatch to sRGB than from sRGB to ColorMatch. The result is a slightly bigger average and a significant increase in the standard deviation for the ColorMatch to sRGB conversions. The higher “visibility” of this effect could explain the popular impression that the ColorMatch space is larger than sRGB.

To place these errors in perspective, we should take into consideration the conditions in which these images will be seen. One of these conditions is the observation time. According to a review article by Has & al.,<sup>28</sup> an inexperienced user will take approximately 5 seconds to notice a  $\Delta E^*_{ab}$  difference of 15 from an original. The time goes up to 10 seconds for a  $\Delta E^*_{ab}$  of 10, and 15 seconds for a  $\Delta E^*_{ab}$  of 5. Another study<sup>29</sup> has shown that errors of less than 2,5  $\Delta E^*_{ab}$  are not visible on real world images shown on a CRT. In essence, the threshold value of  $\Delta E^*_{ab} = 1$  can only be achieved only by prolonged comparative viewing in a controlled environment.

Figure 6: A visual comparison of the ColorMatch and sRGB spaces.





On the hardware side, it has been shown<sup>30</sup> that CRTs require a warm-up time varying between 15 minutes and three hours, depending on models, before achieving a long term stability of  $0,15 \Delta E^*_{ab}$  on average. On a given CRT subjected to a large luminance variation, an initial  $\Delta E^*_{ab}$  of 1,0 was seen to exponentially decrease to about  $0,1 \Delta E^*_{ab}$  in 60 seconds. As for printed material, errors between 2 and  $4 \Delta E^*_{ab}$  are mentioned by Has & al. for the offset and rotogravure process.

From this data we are able to conclude that the procedures shown in these pages can provide accurate results for all but the most critical applications. In all cases however, and in particular for space to space conversions, it is important to verify the extent of clipping since it can result in easily noticeable errors.

## 5 A practical example: the GretagMacbeth ColorChecker

The ColorChecker card is ubiquitous in the photographic and video fields. Its main application is for obtaining a rapid assessment of an imaging devices' calibration, although it can be used for simple calibration purposes. It consists of a small card containing 24 color patches defined to cover common natural colors such as skin colors, foliage, sky, additive and subtractive primaries, and a six steps gray scale.<sup>31</sup> The patches are manufactured for optimum color consistency under varying lighting conditions.

R'G'B' values for this chart are difficult to find. In particular, the R'G'B' coordinates supplied with the card are not properly defined and correspond to none of the spaces presented here. To fulfill this need, the coordinates for twelve R'G'B' spaces are given in Table 11. The source data, xyY coordinates measured with CIE Illuminant C, is from Reference 31.

These coordinates should be used in any program where specific "RGB" values can be assigned. Please notice the absence of primes against the letters of "RGB" in the preceding sentence, which reflect how gamma corrected coordinates are referred to in most software.

When comparing displayed or printed patches with the original set, you may find that there are differences for some or all of the reproduced colors. These differences are most likely due to non-calibrated displays, non-calibrated printers, or wrong printer drivers. Even when using what may seem as the "right" ICC profile, a print may not look perfect. This, in turn, may simply be attributed to non-perfect profiles, a situation which highlights the immature state of these technologies. Although more expensive in terms of process time and hardware requirements, user generated ICC profiles should be used for best results instead of the generic ones supplied by the devices' manufacturers. Procedures to perform this calibration based on the ColorChecker card do exist but most systems generally perform calibration with a larger number of patches, sometimes up in the thousands for high-end applications. Many calibration systems rely on IT8 targets, manufactured by major film companies, that contain 108 standard color patches plus additional neutral and vendor specific patches.



### About the author

Danny Pascale, M.Sc.A., B.Eng.: Following a Bachelor degree in Engineering Physics at École Polytechnique, University of Montreal, he obtained his Master's degree in the same group for the study of non-linear effects in optical fibers. Subsequently, he participated in the development of lasers and instrumentation dedicated to laser-matter interaction research at INRS-Énergie. He then designed electro-optical instrumentation and thermal sensors for military applications at Bendix Avelex, a unit of Allied-Signal Aerospace. A few years later he became a partner at Simdev Electronics, a military simulation system and software design firm where he took charge of video display and positioning systems development. He now does technology assessment and help companies bring new products into the market in the computer and consumer electronics industries. He recently formed a new company, BabelColor, dedicated to the development of colorimetric software tools. He is a member of SPIE, OSA, and IEEE.

No.	Color name	xyY (CIE C)			Adobe			Apple			CIE			ColorMatch			HDTV			NTSC		
		x	y	Y	R'	G'	B'	R'	G'	B'	R'	G'	B'	R'	G'	B'	R'	G'	B'	R'	G'	B'
0	illuminant	0,3101	0,3161	100	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255
1	dark skin	0,4002	0,3504	10,05	107	82	69	95	63	51	112	85	71	93	63	50	103	66	51	92	68	53
2	light skin	0,3773	0,3446	35,82	184	150	132	181	133	112	190	154	134	178	132	111	190	141	119	175	143	122
3	blue sky	0,2470	0,2514	19,33	105	123	156	79	104	142	98	124	153	82	104	143	82	111	149	94	107	146
4	foliage	0,3372	0,4220	13,29	94	109	69	71	91	48	94	104	73	71	91	48	72	95	49	79	97	53
5	blue flower	0,2651	0,2400	24,27	131	129	176	113	110	165	127	134	173	115	109	166	121	117	171	122	113	168
6	bluish green	0,2608	0,3430	43,06	134	190	172	91	180	156	124	182	172	97	180	156	87	184	163	126	184	163
7	orange	0,5060	0,4070	30,05	196	125	52	204	105	25	207	133	67	198	105	22	213	114	25	186	120	43
8	purplish blue	0,2110	0,1750	12,00	81	92	164	58	72	154	69	100	160	63	71	154	60	77	159	70	67	154
9	moderate red	0,4533	0,3058	19,77	173	88	99	178	65	81	182	105	101	173	64	80	190	73	86	162	75	88
10	purple	0,2845	0,2020	6,56	86	60	107	74	42	91	86	70	105	73	41	91	80	42	95	71	37	93
11	yellow green	0,3800	0,4887	44,29	168	188	73	146	178	36	170	179	89	145	179	36	150	182	44	158	188	62
12	orange yellow	0,4729	0,4375	43,06	211	162	58	218	146	22	221	164	77	212	146	20	225	154	27	203	160	51
13	blue	0,1866	0,1285	6,11	55	62	147	35	43	135	36	74	142	41	42	135	32	43	140	43	29	135
14	green	0,3046	0,4782	23,39	100	149	78	61	134	52	96	138	85	64	135	52	52	139	56	86	142	63
15	red	0,5385	0,3129	12,00	154	51	60	158	28	44	164	76	64	152	27	43	170	32	42	141	37	47
16	yellow	0,4480	0,4703	59,10	225	199	56	228	189	0	234	195	83	223	190	0	232	194	13	218	201	51
17	magenta	0,3635	0,2325	19,77	167	85	148	171	61	135	173	107	146	167	60	134	183	69	140	157	65	138
18	cyan	0,1958	0,2519	19,77	64	135	165	0	118	152	26	131	162	17	118	152	0	124	158	56	120	155
19	white 9.5 (.05 D)	0,3101	0,3163	90,01	243	243	243	240	241	240	243	243	243	240	241	240	242	242	242	242	242	242
20	neutral 8 (.23 D)	0,3101	0,3163	59,10	201	201	201	190	190	190	201	201	201	190	190	190	196	196	196	196	196	196
21	neutral 6.5 (.44 D)	0,3101	0,3163	36,20	161	161	161	145	145	145	161	161	161	145	145	145	152	152	152	152	152	152
22	neutral 5 (.70 D)	0,3101	0,3163	19,77	122	122	122	104	104	104	122	122	122	104	104	104	110	110	110	110	110	110
23	neutral 3.5 (1.05 D)	0,3101	0,3163	9,00	85	85	85	67	67	67	85	85	85	67	67	67	70	70	70	70	70	70
24	black 2 (1.5 D)	0,3101	0,3163	3,13	53	53	53	37	37	37	53	53	53	37	37	37	34	34	34	34	34	34

No.	Color name	L*a*b* (CIE D50)			PAL / SECAM			SGI			SMPTE-240M			SMPTE-C			sRGB			Wide Gamut		
		L*	a*	b*	R'	G'	B'	R'	G'	B'	R'	G'	B'	R'	G'	B'	R'	G'	B'	R'	G'	B'
0	illuminant	100	0	0	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255
1	dark skin	38,14	13,81	14,75	102	66	51	76	46	35	103	63	49	104	64	50	116	81	67	104	85	70
2	light skin	66,63	15,38	17,30	188	141	119	168	114	93	192	139	118	192	139	119	197	152	132	179	154	133
3	blue sky	50,73	-3,14	-22,43	83	111	149	61	85	125	77	109	147	78	109	147	96	124	159	108	123	155
4	foliage	43,36	-14,99	21,85	73	95	49	53	72	33	68	94	47	69	95	49	87	109	65	97	105	71
5	blue flower	56,01	9,63	-25,74	121	117	171	94	91	150	119	115	169	119	115	170	133	130	179	131	133	174
6	bluish green	71,50	-31,93	0,83	93	184	163	72	167	139	74	185	162	75	185	162	101	191	172	145	183	172
7	orange	62,28	31,89	58,56	210	114	23	194	86	15	218	110	25	218	111	26	218	126	41	186	135	61
8	purplish blue	40,44	11,42	-44,07	60	77	159	42	54	137	54	73	157	56	74	158	75	91	168	83	97	162
9	moderate red	51,94	45,26	15,56	187	73	86	165	48	63	194	66	84	195	67	85	197	88	100	162	105	101
10	purple	30,49	24,00	-23,65	78	42	96	56	28	72	79	37	93	80	39	94	94	58	109	83	69	106
11	yellow green	72,83	-23,76	58,63	151	182	40	129	165	23	147	184	44	147	184	45	160	190	60	171	181	82
12	orange yellow	72,18	17,41	66,70	222	154	23	210	129	13	229	152	27	229	153	29	228	164	43	204	166	69
13	blue	28,59	20,31	-52,83	33	43	141	22	28	117	27	38	138	28	39	139	48	59	151	56	70	145
14	green	55,66	-38,77	33,09	58	139	55	44	116	36	41	140	55	42	140	56	68	150	72	110	140	81
15	red	41,71	53,43	26,98	167	32	42	142	17	30	174	21	40	175	23	42	179	47	58	142	76	63
16	yellow	81,95	1,66	78,47	231	194	5	223	177	0	235	194	16	235	195	18	235	201	28	221	197	71
17	magenta	51,56	49,00	-15,57	180	69	141	156	44	117	187	61	139	187	62	139	190	84	151	157	106	147
18	cyan	51,06	-28,00	-27,36	0	124	158	0	99	135	0	123	157	0	124	157	0	136	168	82	130	163
19	white 9.5 (.05 D)	96,00	-0,05	0,06	242	242	242	237	237	237	242	242	242	242	242	242	243	244	243	243	243	243
20	neutral 8 (.23 D)	81,35	-0,04	0,06	196	196	196	178	178	178	195	195	195	195	195	195	202	202	202	201	201	201
21	neutral 6.5 (.44 D)	66,67	-0,04	0,05	152	152	152	128	128	128	151	151	151	151	151	151	162	162	162	161	161	161
22	neutral 5 (.70 D)	51,58	-0,03	0,04	110	110	110	85	85	85	108	108	108	109	109	109	123	123	123	122	122	122
23	neutral 3.5 (1.05 D)	35,98	-0,02	0,03	70	70	70	50	50	50	67	67	67	69	69	69	85	85	85	85	85	85
24	black 2 (1.5 D)	20,56	-0,02	0,02	34	34	34	24	24	24	31	31	31	33	33	33	49	50	49	53	53	53

Table 11: The GretagMacbeth ColorChecker patches color coordinates for the RGB spaces defined in Table 5. Whenever applicable, a detailed gamma function was used in the conversions. The xyY coordinates are from Reference 31, as determined with CIE Illuminant C. L\*a\*b\* coordinates are given for CIE Illuminant D50.

## References:

- <sup>1</sup> R.W.G. Hunt, *The reproduction of Colour*, 5<sup>th</sup> ed., Fountain Press (1995), ISBN 0-86343-381-2.
- <sup>2</sup> *Les mécanismes de la vision*, préface d'Yves Galifret, Pour La Science 1979 à 1989, ISBN 2-9029-18-43-7.
- <sup>3</sup> *The Science of Color*, Committee on Colorimetry, Optical Society of America, Washington (1973), ISBN 0-96003-801-9.
- <sup>4</sup> ASTM E308-99: "Standard Practice for Computing the Colors of Objects by Using the CIE System," available from their Web site: <http://www.astm.org>.
- <sup>5</sup> From files originally contributed by Mike Vrhel. Available by ftp on NCSU's Electrical and Computer Engineering department Web site: <ftp://ftp.eos.ncsu.edu/pub/spectra/>.
- <sup>6</sup> ColorChecker, product No. 50105, manufactured by Munsell Color, division of GretagMacbeth.
- <sup>7</sup> W. Thornton, "Matching lights, visual response, and the painfully sub-human CIE Standard Observers," *Perceiving, Measuring and Using Color*, Vol. 26, 1990, p. 1250.
- <sup>8</sup> Uwe Brinkmann, "Solid-state lasers project laser television signals," *Laser Focus World*, November 1997, pp. 52-56.
- <sup>9</sup> David Hargis, Allen Earman, "Lasers replace conventional technology in display designs," *Laser Focus World*, July 1998, pp. 145-149.
- <sup>10</sup> G.A. Agoston, *Color Theory and its Application in Art and Design*, Springer Series in Optical Sciences, Vol. 19, David L. MacAdam ed., Springer-Verlag, Berlin (1979), ISBN 3-540-09564-X.
- <sup>11</sup> David L. MacAdam, "Visual sensitivities to color differences in daylight," *J. Opt. Soc. Am.*, Vol. 32, 1942, pp. 247-273.
- <sup>12</sup> MIL-HDBK-141, Section 4.
- <sup>13</sup> *The Infrared & Electro-Optical Systems Handbook*, Vol. 1: *Sources of Radiation*, George J. Zissis ed., Joseph S. Accetta & David L. Shumaker executive ed., ERIM & SPIE (1993), ISBN 0-8194-1072-1.
- <sup>14</sup> ICC Specification ICC.1:1998-09: "File Format for Color Profiles," available on the Internet at <http://www.color.org>.
- <sup>15</sup> M.R. Luo, R.W.G. Hunt, B. Rigg, K.J. Smith, "Recommended colour-inconstancy index," *Journal of the Society of Dyers and Colourist*, Vol. 115, May/June 1999, pp. 183-188.
- <sup>16</sup> Mary Nielsen, Michael Stokes, "The Creation of the sRGB ICC Profile," *Proceedings of IS&T Sixth Color Imaging Conference: Color Science Systems and Applications 1998*, Scottsdale, Arizona, ISBN 0-89208-213-5.
- <sup>17</sup> Roy S. Berns, Ricardo J. Motta, Mark E. Gorzynski, "CRT Colorimetry, Part I: Theory and Practice," *COLOR research and application*, Vol. 18, No. 5, Oct. 1993, pp. 299-314.
- <sup>18</sup> Charles A. Poynton, *A Technical Introduction to Digital Video*, John Wiley & Sons (1996), ISBN 0-471-12253-X. From the same author, on the Internet, on the site <http://www.poynton.com/Poynton-color.html> :  
" Frequently Asked Questions About Gamma," available as "GammaFAQ.pdf";  
" The rehabilitation of gamma," available as "Rehabilitation\_of\_gamma.pdf";  
" Frequently Asked Questions About Color," available as "ColorFAQ.pdf".
- <sup>19</sup> PNG (Portable Network Graphics) Specification, Version 1.0, World Wide Web Consortium (W3C) Recommendation 01-October-1996, available on the Internet on the site <http://www.w3.org>.
- <sup>20</sup> J. R. Jiménez, J. F. Reche, J. A. Díaz, L. Jiménez del Barco, E. Hita, "Optimization of Color Reproduction on CRT-Color Monitors," *COLOR research and application*, Vol. 24, No. 3, June 1999, pp. 207-213.
- <sup>21</sup> From Reference 17. The gamma function of that reference is defined with a different equation than the one used in this document. The gamma curve of the reference was regenerated and fitted against the detailed gamma function defined in this document.
- <sup>22</sup> Karl Lang, miro displays inc., private communication, 1999.
- <sup>23</sup> ANSI : <http://www.ansi.org> ; EBU : <http://www.ebu.ch> ; IEC : <http://www.iec.ch> ; ITU : <http://www.itu.int> ; SMPTE: <http://www.smpte.org>.
- <sup>24</sup> SMPTE Recommended Practice RP 177-1993: "Derivation of Basic Television Color Equations."
- <sup>25</sup> Ján Morovič, *To develop a Universal Gamut Mapping Algorithm*, Ph.D. thesis, University of Derby, October 1998, available on the Internet at <http://colour.derby.ac.uk/%7Ejan/pubs/thesis.html>.
- <sup>26</sup> Alison Gilchrist, Jim Nobbs, "Colour by numbers," *Journal of the Society of Dyers and Colourist*, Vol. 115, Jan. 1999, pp. 4-7, available on the Internet at <http://www.sdc.org.uk/pdf-features/jn99f1.pdf>.
- <sup>27</sup> The method is described in Reference 16. The results are from the slide presentation at the conference.
- <sup>28</sup> Michael Has, Todd Newman, "Color Management: Current Practice and the Adoption of a New Standard," available on the Internet on the ICC Web site at <http://www.color.org/wpaper1.html>.
- <sup>29</sup> Michael Stokes, M.D. Fairchild, Roy S. Berns, "Colorimetric quantified visual tolerances for pictorial images," *Comparison of Color Images Presented in Different Media*, *Proceedings 1992*, Vol. 2, M. Pearson ed., Tech. Assoc. Graphic Arts and Inter-Soc. Color Council, pp. 757-777.
- <sup>30</sup> Roy S. Berns, Mark E. Gorzynski, Ricardo J. Motta, "CRT Colorimetry, Part II: Metrology," *COLOR research and application*, Vol. 18, No. 5, Oct. 1993, pp. 315-325.
- <sup>31</sup> C.S. McCamy, H. Marcus, J.G. Davidson, "A Color-Rendition Chart," *J. Appl. Phot. Eng.*, Vol. 2, No. 3, Summer 1976, pp. 95-99, Society of Photographic Scientists and Engineers (now called "The Society for Imaging Science and Technology"; Web site: <http://www.imaging.org>).