A comparison of four multimedia RGB spaces

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Abstract. Four RGB spaces in common usage for consumer and professional computer graphics, and for multimedia applications are compared by analyzing the clipping statistics when converting randomly selected colors from one space to another. It is shown that comparing their gamut sizes and assessing the gamut overlap by using only the simple triangular shape define by their primaries on the chromaticity diagram can lead to large errors. It is shown that even for spaces of very similar size located close to one another, a large number of conversions are clipped because the regions near the edges of the gamut correspond to densely packed, gamma compressed, colors.

Subject terms: colorimetry, computer graphics, color, RGB, color space, color gamut

1. Introduction

This article is concerned about RGB spaces used in computer graphics and, by extension, multimedia applications. RGB spaces are the territories of fierce commercial competition opposing hardware and software companies, and a subject of debate amongst trade specialists. One industry goal is to provide an organized, ideally seamless, color management workflow for content creation and display. The context is an Internet driven world where TVs and computers converge and where it can now be more clearly seen that it is the supporting technologies that will converge, and not the applications.

In the 1970’s, “computer graphics” for TV use were conceived on dedicated systems. Low cost Personal Computers (PCs) now generate most of this content even though the RGB space of a computer display can be quite different from the one of a TV. In many cases nowadays the color space conversion is handled by the “video-out” portion of the computer’s graphic card without much user control.

Very recently, the cross-platform acceptance of the International Color Consortium (ICC) color profiling method helped bring uniformity to the picture, at least in the computer portion of the field. With ICC profiles of their input, output and display devices, vendors can transparently exchange color data with other vendors’ products. The color data conforms to well characterized color spaces and guarantees a minimal color appearance uniformity.

While Digital TV (DTV) and High-Definition TV (HDTV) sets are available to consumers, standards are still being actively developed. In particular, the sRGB standard (IEC 61966-2-1), based on ITU-R BT.709-3, a standard for HDTV, is proposed as a solution to dual-use PC/TV images and video content for most consumer applications.

ICC profiling and standard spaces such as sRGB are the two major options proposed to handle content creation and diffusion. They do not target the same applications and users but they do overlap in many areas. For example, sRGB, for which Hewlett-Packard and Microsoft are the champions, is proposed as a computer-efficient means of dealing with images since, by making all media content sRGB compatible, no profile embedding and data conversion are required in the input, output and processing devices. This slimmed-down file format is particularly adapted to an Internet environment with high image content and limited communication bandwidth. Also, the sRGB standard was devised to be compatible with legacy content (both IBM-compatible PCs, and TVs).

The ICC proponents are more concerned with higher color fidelity for professional users and applications for color-conscious consumer, and, in particular, the applications dedicated to the printed media. Nonetheless, both options consider multimedia uses and the Internet as crucial to support.

The debate around the two approaches was crystallized when Version 5 of the Adobe Photoshop image editing software was commercialized. The color management workflow tools it contained had “default” selections for the working RGB space in which these files would now be edited, and for the legacy files saved without ICC profiles. These assumptions were severely criticized. So much so that a more flexible interface was soon offered as Version 5.0.2 and now this application, in its latest releases, is considered a forerunner of how color management should be handled.

With the ability to work in any RGB space and the capability to transform the color content between one space and another, the debate has now shifted on which RGB space is the best, and which one has the largest gamut. In most trade magazines, the ColorMatch and Apple RGB spaces are considered better, and by extension larger, than the sRGB space, with an hedge for the ColorMatch space. This conclusion is sometimes reached by graphically comparing, on a CIE 1931 chromaticity diagram, the xy coordinates of the display space primaries to the primaries of common printed media inks. The problem with this approach is that the gamut size is only loosely related to the area in the xy plane. Also, the conclusion may vary depending on the selection of the
printing inks, particularly with the newer extended gamut ink sets using six or more base inks.

This paper intends to provide an analysis of the relative sizes of these spaces by comparing a high number of randomly selected color transforms between two spaces, and between the three similar size spaces against a significantly larger space, Adobe RGB (1998), proposed for professional printing applications.

2. The four color spaces

Defining a color space is a compromise between the availability of good primaries – of inks, monochromatic laser wavelengths, phosphors –, the signal to noise ratio of the real-world display and signal processing devices, and the number of digital levels that can be effectively managed by the computing devices. There is no point in defining a very large gamut if the number of possible colors that can be coded is so small that the eye will see discrete steps – banding – where uniform gradients are required, a typical problem of digital systems, or if the color space gamut is much bigger than the gamut of all output devices. Assigning more bits to each primary is a solution which is now seen more often; it requires more computing power but it minimizes banding due to repetitive image correction and manipulation even if the final image is down-sampled to be compatible with the range of the output device.

Because there is still a large quantity of software which is not able to cope with image files embedded with color calibration profiles, either for competitive or practical reasons, the spaces associated to computing platforms are usually defined relative to a specific reference display. For example, the Apple RGB space is defined with Sony Trinitron phosphors, even though other Apple products, like the iMac, use a Cathode Ray Tube (CRT) from another manufacturer with different characteristics. In most modern operating systems (Mac OS 9 and X, Windows 98, Me, 2000, and XP), display calibration is handled by the operating system independently of the file’s color space to account for these differences.

RGB spaces have evolved, sometimes for technological reasons (NTSC to SMPTE-C), sometimes to fulfill professional requirements (ColorMatch, Adobe RGB), and sometimes because that’s how the display was built and it became a de-facto, standard (Apple RGB). A short description of the four RGB spaces considered in this study follows. Numerical specifications are shown in Table 1. The position of their primaries on a CIE 1931 chromaticity diagram can be seen in Figure 1.

Adobe RGB (1998)

Formerly known as SMPTE-240M for Photoshop user, this space has been renamed once the final SMPTE-240M standard committee settled for a smaller gamut. Adobe RGB is very close to the original NTSC space and has a large enough gamut that encompasses the gamuts of most printing processes and displays. However, because of its size, a 16 bit per primary content creation and editing system should be preferred to an 8 bit per primary format. While a relatively large number of colors cannot be printed using the SWOP process (SWOP: Specifications for Web Offset Publications), particularly in the green portion of the gamut, newer printing processes, such as Pantone Hexachrome, take advantage of this space.

Apple RGB

A very common RGB space on the desktop, its gamut size is similar to the ones of the ColorMatch and sRGB spaces. The Apple RGB, like the ColorMatch and SGI spaces, has a non-unity display lookup-table (LUT) gamma which is compensated by the file encoding gamma (see Section 4 for a discussion of gamma). When a value of 1,8 is entered by the user in the control panel for display gamma, the LUT is filled with numbers corresponding to a gamma equal to 1,8/2,6=0,69 (or 1,45 if you define gamma using the reciprocal value =1/0,69). ColorSync, Apple’s color management technology at the operating system level, automatically takes care of color calibration for all input and output devices.
output devices and can also be used to convert files from one space to another.

**ColorMatch RGB**
This space was originally devised by Radius (now Digital Origin) to be used in conjunction with its PressView line of calibrated displays dedicated to professional use.9 Because of its calibrated environment, it is often favored over other desktop spaces by the print industry. Compared with sRGB, it has a slightly larger gamut in the blue-green region but a smaller one in the red-blue region.

**sRGB and HDTV RGB**
Identical in terms of gamut, these two spaces differ only in their definition of the viewing conditions, which are simply assumed in ITU-R BT.709-3, a High-Definition-TV (HDTV) standard, and precisely defined in IEC 61966-2-1, the sRGB standard. With chromaticities not very far from SMPTE-C (and SMPTE-240M), they strive to represent the evolution of the standard North-American TV and its convergence with the PC world, while maintaining compatibility with the large quantity of recorded media.

An extended gamut color encoding standard has been proposed for sRGB10; it supports multiple levels of precision while being compatible with the base standard.

### 3. Comparison method
We compare the four spaces by statistically evaluating the outcome of the conversion of randomly selected samples from one space to the other. This procedure is obviously not very precise to determine relative size for spaces which are similar in dimensions and overlap partially; nonetheless it enable us to evaluate the proportion of colors clipped and determine the error distribution. A better evaluation of their relative size can be obtained by evaluating transform statistics from the larger Adobe RGB (1998) to all three others, which it contains – with a small caveat for the ColorMatch space.

Converting R′G′B′ triads from one space to another is performed with the following sequence:

- a) R′G′B′, to RGBs;
- b) RGBs, to XYZ;
- c) XYZ, to XYZd (if not the same Illuminant);
- d) XYZd to RGBd;
- e) RGBd to R′G′B′d;

where the source and destination spaces have an “s” and “d” subscript respectively. Step “a)” converts the gamma corrected R′G′B′ data to linear RGB values. This step and its inverse procedure, step “c)”, are discussed in Section 4.

In step “b)”, linear RGB data is converted to tristimulus XYZ values defined according to the CIE 1931 standards. This step and its associated inverse procedure, step “d)”, are presented in Section 5.

Step “c)” is required if the source and destination spaces are not based on the same illuminant. A simplified Bradford matrix transform is used for this task; it is presented in Section 6.

### 4. Gamma
The eye is more sensitive to variations of luminance in low luminance levels than similar variations in high luminance levels. R′G′B′ values, commonly referred to by “RGB” in most application software, are scaled according to this non-linear perception of the eye and more data triads are assigned to the lower luminance levels. As a result, the R′G′B′ scale is close to a perceptively linear scale where doubling the values of a triad will result in a color whose brightness appears doubled.

The use of the word gamma for this compression process is an element of discord. Originally coined to explain the straight-line portion of the S-shaped (sigmoid) curve obtained...
when tracing, on log-log scales, the optical density of photographic film in relation to exposure, the so-called H&D curve from its inventors Hurter and Driffield, it has been since misused and overused. Some authors propose the more generic term “exponent” instead. We will nonetheless use the term gamma in this paper since it is associated with fundamental aspects of display technology and human perception, to which a generic term like “exponent” would not do justice. However, you should always verify how gamma is defined before making comparisons with other sources of information, and you should get used to the fact that any author’s gamma value could be the reciprocal of another author’s definition.

A comprehensive presentation of modern CRT characteristics is contained in a paper by Berns, Motta and Gorzynski. A very thorough discussion of gamma can be found in the book and the Internet articles of Poynton. The definition of the various flavors of gamma is well presented in a tutorial that is part of the Portable Network Graphics (PNG) Specification published by the World Wide Web Consortium (W3C).

A typical vision chain includes:

i- A file gamma that combines the camera gamma and the software-encoding gamma (γfile = γcamera \* γencoding). In this document we will consider that the camera gamma and the encoding gamma are defined by the same equation, that only one of them is used at one time, and that they simply distinguish the origin of the data, either a camera or a software program.

ii- A decoding gamma, which is defined as the gamma of any transformation performed by the software reading the image file. In this document we will assume that the software does not modify the gamma once the original file is created and that the decoding gamma is equal to one.

iii- A display gamma, which combines the LUT gamma and the CRT gamma: (γdisplay = γLUT / γCRT).

iv- The overall gamma that combines all the preceding gammas.

v- The human eye gamma.

File gamma - The effect of camera gamma is often defined in the form:

\[ V = (1 + \text{offset})L' - \text{offset} \quad \text{for } 1 \geq L \geq \text{transition} \]

\[ V = \text{slope} \times L \quad \text{for } \text{transition} > L \geq 0 \quad (1) \]

where L is the image luminance (0 ≤ L ≤ 1) and V is the corresponding electrical signal (in Volt). An example of the values found for the offset, gamma, transition and slope parameters in ITU-R BT.709-3 are:

\[ \text{offset} = 0.099 \]
\[ \gamma = 0.45 \]
\[ \text{transition} = 0.018 \]
\[ \text{slope} = 4.5 \]

The function is defined by two segments: a linear segment at low light levels, below the defined transition level, which makes the transform less susceptible to noise around zero luminance, and a power segment with a 0.45 exponent. As mentioned before, the effect of that exponent is to compress the luminance signal by assigning a larger signal range to dim colors, where the eye is most sensitive, and a small signal range to bright colors.

The offset term of Equation (1) is related to what is generally identified in TVs and monitors as the black level, intensity or brightness control knob. The combination of (1 + offset) is related to the picture, gain or contrast knob. It may sound surprising that brightness be associated with a DC level and contrast to a term which controls the maximum luminance level, but these terms were defined in relation to what is perceived, not the mathematical expression. In effect, the eye perceives as a brightness increase a change in the black level more than it does of a change in the gain. Note: in some displays, the brightness and contrast knobs are effectively labeled the reverse of what is “generally” found!

Equation (1) can be approximated by a simpler function of the form:

\[ V = L' \quad \text{for } 0 \leq L \leq 1 \quad , \quad (2) \]

with a gamma optimized to fit the data of the detailed transform. Taking ITU-R BT.709-3 again as an example, a best-fit curve can be obtained with the simpler form of Equation (2) and a gamma of 0.519. The simpler form is often retained to improve computing efficiency in software applications. When defined, we used the detailed function.

For software generated files, it is customary to apply a simple gamma correction of the form described in Equation (2) with an exponent value that is different between computing platforms. As shown in Table 1, this exponent is usually 0.455 (1/2.2) for sRGB – a space used only in Windows based computers as this text is written. It is 0.56 (1/1.8) for Macintosh. The luminance “L” in Equation (2) corresponds, and is linearly proportional, to either one of the R, G or B channels values.

The voltage “V” corresponds to the “gamma corrected” coordinates R’, G’, or B’, the values shown in graphic software dialog boxes as “R”, “G”, and “B”. Depending on your choice of a detailed or simple gamma, R’, G’, and B’ are determined with either one of the following equations (for simplicity, only R’ is shown; G’ and B’ are similar; R, G, and B have to be normalized between 0 and 1 prior to this operation):
\[ R' = \text{round}\left(255 \times \left(1 + \text{offset}\right)R' - \text{offset}\right) \]

for \(1 \geq R' \geq \text{transition}, \text{and} \)

\[ R' = \text{round}\left(255 \times \text{slope} \times R\right) \]

for \(\text{transition} > R' \geq 0\) ,

\( (3) \)

or:

\[ R' = \text{round}\left(255 \times R'^{\gamma}\right) \quad \text{for } 0 \leq R' \leq 1 \]  \( (4) \)

These equations, to be used for step “c)”, defined in Section 3, are similar to Equations (1) and (2) with terms added to scale and round the values to the nearest integer between zero and 255. This scale corresponds to 8 bits per primary, a 24-bit color system.

The reverse equations, corresponding to step “a)”, are \((R', G', B')\) have to be normalized between 0 and 1 prior to this operation:

\[ R = 255 \times \left(\frac{(R+\text{offset})}{(1+\text{offset})}\right)^{1/\gamma} \]

for \(1 \geq R' \geq \text{(transition x slope)}, \) and

\[ R = 255 \times R'/\text{slope} \]

for \(\text{(transition x slope)} > R' \geq 0\) ,

\( (5) \)

or

\[ R = 255 \times R'^{1/\gamma} \quad \text{for } 0 \leq R' \leq 1 \]  \( (6) \)

**Display gamma** - In Windows type PCs, the graphics card LUT is nominally a straight-line one-to-one transfer function. In Apple’s Macintosh, the graphics card LUT has a transfer function as per Equation (2) with the exponent being 0.69 \((1/1.45)\). It just so happens, and it should not be surprising, that the value of \(\gamma_{\text{LUT}}\) is very similar for all platforms.

In many TV standards, a reference reproducer, which corresponds to an idealized display, is expressed in a form which is the reverse of the camera transfer function shown as Equation (1), and essentially the same as Equation (5):

\[ L = \left(\frac{(V + \text{offset})}{(1 + \text{offset})}\right)^{1/\gamma} \]  \( (7) \)

There again, a simpler, approximate, transfer function can be written:

\[ L = V^{1/\gamma} \]  \( (8) \)

In practice, however, the camera and display gammas are different so that the displayed contrast is higher than the original image contrast. This is done because in dim or dark ambient conditions, a frequent condition for TV viewing, dark tones are perceived brighter than they should and the black to white contrast is lower. Assuming that \(\gamma_{\text{encoding}}\) and \(\gamma_{\text{LUT}}\) are equal to one, a normal assumption for TV work, the ratio between the camera and CRT gammas is typically fixed to 1.25 for dim viewing conditions.\(^{14}\)

In a properly set monitor for color related work, it is recommended to adjust the black level, or offset, near zero – i.e. barely perceptible from a no-signal state. Also, it is recommended to adjust the video gain – contrast – to maximum value. This is the method used in the Adobe Gamma “Control Panel” tool provided with many Adobe products, and a recent paper by J. R. Jiménez & al.\(^{15}\) confirms that this procedure maximizes the color gamut.

Berns & al.\(^{16}\) present results of measurements taken on properly set monitors that are best fitted, when using Equation (7), with a gamma of 0.406 \((1/2.46)\) and an offset of zero. In this case Equation (7) corresponds exactly to Equation (8). A rounded value of 0.4 \((1/2.5)\) is used as a generic CRT gamma in Table 1.

**Overall gamma** - The overall system gamma is:

\[ \gamma_{\text{overall}} = \frac{\gamma_{\text{file}} \times \gamma_{\text{LUT}}}{\gamma_{\text{CRT}}} \]  \( (9) \)

It can be seen in Table 1 that the overall gamma varies between 0.96 and 1.14, somewhat lower values than the 1.25 ratio usually expected for TV viewing. This result is consistent with the brighter illumination conditions typical of computer work and the correspondingly higher, in fact more normally, perceived contrast. At some point however, veiling glare could lower the contrast again. This explains why professional systems have glare protecting hoods around monitors, as well as neutral gray bezels – and sometimes an entirely gray workplace – to prevent unwanted color contamination.

**Human eye gamma** - The human eye has a response similar to the one assigned for cameras. In the \(L^*a^*b^*\) color space, one of the “more” uniform color spaces standardized by the CIE\(,\) the perceived luminance \(L^*\), called lightness, is essentially the same as Equation (1) but with a 0.33 \((1/3)\) exponent.

The \(L^*a^*b^*\) is derived from the \(XYZ\) data with the following transform (also from Ref. 14):

\[ L^* = 116\left(\frac{Y}{Y_n}\right)^{1/3} - 16 \quad \text{for } \frac{Y}{Y_n} > 0.008856 \]

\[ L^* = 903.3\left(\frac{Y}{Y_n}\right) \quad \text{for } \frac{Y}{Y_n} \leq 0.008856 \]

\[ a^* = 500\left(\frac{X}{X_n}\right)^{1/3} - \left(\frac{Y}{Y_n}\right)^{1/3} \]

\[ b^* = 200\left(\frac{Y}{Y_n}\right)^{1/3} - \left(\frac{Z}{Z_n}\right)^{1/3} \]  \( (10) \)
The camera signals, or encoded file data if the image is generated directly in software, are thus compressed in an efficient way with more signal range associated with the lower brightness colors where the eye has more discrimination. To be viewed, the image goes through the graphics LUT and the CRT electronics, a path that effectively decompresses the recorded signal so that the eye can perceive it as if he saw the original scene, with a more or less serious correction added to account for viewing conditions.

5. From RGB to XYZ, and vice-verse

The XYZ to RGB matrices for the four spaces discussed in this article were determined according to the recommended practice RP 177-93 from the Society of Motion Picture and Television Engineers.17 The matrices are shown in Table 1.

The XYZ triads are obtained with the following multiplication, with RGB values normalized to 100:

\[
\begin{bmatrix}
X \\
Y \\
Z \\
\end{bmatrix} = \begin{bmatrix}
RGB \rightarrow XYZ \\
\cdot \\
\cdot \\
BG \\
\end{bmatrix} \begin{bmatrix}
R \\
\cdot \\
\cdot \\
G \\
\end{bmatrix} \begin{bmatrix}
\cdot \\
\cdot \\
\cdot \\
B \\
\end{bmatrix}. \quad (11)
\]

Similarly, the RGB triads are obtained with the following multiplication, with Y normalized to 100:

\[
\begin{bmatrix}
R \\
G \\
B \\
\end{bmatrix} = \begin{bmatrix}
XYZ \rightarrow RGB \\
\cdot \\
\cdot \\
BG \\
\end{bmatrix} \begin{bmatrix}
XYZ \\
\cdot \\
\cdot \\
ILL \\
\end{bmatrix} \begin{bmatrix}
X \\
\cdot \\
\cdot \\
Y \\
\end{bmatrix} \begin{bmatrix}
\cdot \\
\cdot \\
\cdot \\
Z \\
\end{bmatrix}. \quad (12)
\]

After this operation, the RGB coordinates of the illuminant are (100, 100, 100). All RGB triads should be rescaled at this point – divided by 100 – with the resulting RGB “white” coordinates of (1, 1, 1). Results over one or below zero are clipped at one and zero respectively.

Using both the RGB-to-XYZ and XYZ-to-RGB matrices, we can transform RGB data from one RGB space to another. If the illuminant is not the same for both spaces, we need to apply an illuminant – chromatic – adaptation transform in mid process. One such transform is the simplified Bradford matrix transform presented in the next section. The RGB space-to-space conversion procedure is then represented by the equation:

\[
\begin{bmatrix}
R \\
G \\
B \\
\end{bmatrix}_{\text{space}} = \begin{bmatrix}
XYZ \rightarrow RGB \\
\cdot \\
\cdot \\
BG \\
\end{bmatrix} \begin{bmatrix}
X \\
\cdot \\
\cdot \\
Y \\
\end{bmatrix} \begin{bmatrix}
\cdot \\
\cdot \\
\cdot \\
Z \\
\end{bmatrix} \begin{bmatrix}
\cdot \\
n \cdot \\
\cdot \\
\cdot \\
\end{bmatrix}
\]

If the illuminant is the same, the Bradford matrix is simply omitted. It is important to mention that converting from one space to another is frequently performed in conjunction with an additional step, called gamut mapping, which is not represented in the preceding equation. Gamut mapping algorithms attempt to minimize the effects of clipping by distorting the values of either or both the clipped and non-clipped colors. Variants of the process, still a subject of active research,18 have been devised for different requirements such as maintaining saturated colors in business graphics or achieving a balanced “realistic” look in pictures, even if none of the resulting colors are accurate. However, it should be pointed out that most RGB to RGB conversion matrices found in the literature are simply the RGB-to-XYZ and XYZ-to-RGB matrices of Equation (13) combined into one, as per ASTM RP 177-93, with no Bradford matrix or gamut mapping.

6. The Bradford Matrix

The colorimetric data of a sample cannot be dissociated from the characteristics of the illuminant. In the ideal case, obtaining the colorimetric coordinates of the sample under another illuminant requires reprocessing the spectral data of the sample with the spectral characteristics of the illuminant. However, this computer intensive process is not efficient and requires a large spectral database for each color. But more importantly, for most applications, like image processing, spectral data is simply not available.

To ease this task, chromatic adaptation transforms that transform colorimetric information from their XYZ coordinates have been devised. All modern color appearance models competing for international acceptance19 incorporate such a transform. One contender that has withstood critical revue is called the Bradford, or BFD for short, chromatic adaptation transform.

A simplified matrix representation of the Bradford transform was found to give excellent results during the work performed in the development of the sRGB standard.20 In its simplified version, the only data required to generate the Bradford matrix are the XYZ coordinates of the source and destination whites. The source white is the illuminant used to measure the original data, and the destination white is the illuminant to which the data has to be translated. The Bradford conversion matrix is derived with the three following relations:

\[
\begin{bmatrix}
R_{sw} \\
G_{sw} \\
B_{sw} \\
\end{bmatrix} = \begin{bmatrix}
0,8951 & 0,2664 & -0,1614 \\
-0,7502 & 1,7135 & 0,0367 \\
0,0389 & -0,0685 & 1,0296 \\
\end{bmatrix} \begin{bmatrix}
X_{sw} \\
Y_{sw} \\
Z_{sw} \\
\end{bmatrix}. \quad (14)
\]

\[
\begin{bmatrix}
R_{dw} \\
G_{dw} \\
B_{dw} \\
\end{bmatrix} = \begin{bmatrix}
0,8951 & 0,2664 & -0,1614 \\
-0,7502 & 1,7135 & 0,0367 \\
0,0389 & -0,0685 & 1,0296 \\
\end{bmatrix} \begin{bmatrix}
X_{dw} \\
Y_{dw} \\
Z_{dw} \\
\end{bmatrix}. \quad (15)
\]
where \((\text{RGB})_{dw}\) and \((\text{XYZ})_{dw}\) are the coordinates of the destination white, and \((\text{RGB})_{sw}\) and \((\text{XYZ})_{sw}\) are the coordinates of the source white. In Equations (14), (15) and (16), the 3x3 matrix, with "0,8951" as its top-left element, is called the cone response matrix. In Equation (16), the 3x3 matrix, with "0,9870" as its top-left element, is called the inverse cone response matrix. These two matrices are, as their name says, the inverse of one another.

\((\text{RGB})_{sw}\) and \((\text{RGB})_{sw}\) are first calculated with Equations (14) and (15). \(\text{XYZ}\) coordinates can be derived from the \(xy\) coordinates of Table 1 with the following equations (\(Y = 100\) by definition):

\[
X = x(Y/y) \quad \text{and} \quad Z = (1-x-y)(Y/y).
\]  

The Bradford matrix is then determined from Equation (16) using the ratios of the previous calculations. Bradford matrices for the two illuminants used in the four spaces are shown in Table 2.

Using the Bradford matrix, the XYZ coordinates corresponding to the illuminant of the target RGB space are:

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_{\text{dest.}} = \begin{bmatrix}
0.9870 & -0.1471 & 0.1600 \\
0.4323 & 0.5184 & 0.0493 \\
-0.0085 & 0.0400 & 0.9685
\end{bmatrix} \cdot \begin{bmatrix}
0 & 0 & 1 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_{\text{source}}.
\]  

Table 2: Bradford Matrices between the standard CIE 1931 \(D_50\) and \(D_65\) illuminants.

### 7. Conversion process accuracy

Color differences can be expressed mathematically for any space but they make practical sense only for the more uniform spaces where the resulting numbers can be better associated to what the eye perceives.

For the \(L^*a^*b^*\) space the color difference equation is, again from Ref (11):

\[
\Delta E_{ab}^* = \left[ (\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2 \right]^{1/2},
\]  

where \(k=1\) for samples compared in close proximity (\(k=0,5\) or less for samples compared further away from each other, where the eye is less sensitive to lightness differences). \(\Delta E_{ab}^*=1\) corresponds to colors which are barely differentiable by 50% of a group of observers; the other 50% would see no difference. Even though Equation (20) is a workhouse of the color industry, its statistical threshold is a cause of concern, and of possible litigation, in many industrial applications where expert observers’ judgments are confronted. For this reason, better color difference equations are being sought.

When converting from one space to another, beside the inherent errors coming from the accuracy of the original data, the conversion process can introduce additional errors from the number of decimal places used in the conversion matrices constants, from the approximate form of the Bradford matrix, from the clipping required to limit RGB values between zero and one, from the use of a simple gamma instead of a detailed gamma, and from the rounding of the \(R'G'B'\) values. Table 3 shows typical errors associated with each operation.

<table>
<thead>
<tr>
<th>Processing step</th>
<th>Average (\Delta E_{ab}^*) error</th>
<th>Standard deviation</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bradford matrix</td>
<td>1,4</td>
<td>0,9</td>
<td>Measured for a (D_65) to (D_50) conversion. From Reference 27.</td>
</tr>
<tr>
<td>XYZ to RGB (matrix)</td>
<td>(\approx 0)</td>
<td>(\approx 0)</td>
<td>Negligible error when constants with at least 4 significant decimals are used.</td>
</tr>
<tr>
<td>XYZ to RGB (clipping)</td>
<td>variable</td>
<td>variable</td>
<td>See text.</td>
</tr>
<tr>
<td>RGB to (R'G'B') (simple vs. detailed gamma)</td>
<td>1,3</td>
<td>0,92</td>
<td>When a simple gamma expression is used instead of a detailed one (when available). Measured for sRGB.</td>
</tr>
<tr>
<td>RGB to (R'G'B') (rounding error)</td>
<td>0,23</td>
<td>0,11</td>
<td>Typical values. Values are slightly higher for larger spaces (Ex.: 0,28 average for Adobe (1998)).</td>
</tr>
</tbody>
</table>

Table 3: Typical errors associated to a XYZ to \(R'G'B'\) conversion. Errors due to clipping are not considered.
A detailed evaluation of the Bradford matrix accuracy was performed on over 1 000 colors from the Pantone color data set covering a very large gamut.22 A first set of color coordinates was determined from spectral data and the D65 illuminant with a method similar to the one described in ASTM E308-99.23 A second set of coordinates was obtained by converting XYZ data, obtained from spectral data and with Illuminant D50, to Illuminant D65 using the simplified Bradford matrix. The average $\Delta E^*_{ab}$ error between the two sets was 1.4 with a standard deviation of 0.9.

The error associated with the Bradford matrix presented above does not include any effect resulting from the precision of the matrix terms. If constants with at least four significant decimals are used, then virtually no error is induced by the conversion. This is also true for the XYZ to RGB matrix.

Clipping error values are not shown in this table since they are very dependent of the specific target space and the gamut of the original data. Clipping will most often be noticed for images which exhibit single-color large-area zones, an annoying situation if that color is associated with a “brand” product. This is where the use of “spot” colors – additional inks other than CMY – is justified in many graphic design applications.

Using a simple gamma expression when a detailed one is available adds a $\Delta E^*_{ab}$ of 1.3 on average with a standard deviation of 0.92, about the same error as for the Bradford matrix.

Rounding the R’G’B’ introduces an inevitable error of 0.23, on average, which will not be noticed by most viewers in most viewing conditions. However, multiple conversions between different RGB spaces could degrade the color fidelity to a point where it could be easily perceived.

The errors of Table 3 should not be added since they are statistical in nature. The combined effect of multiple processes can be evaluated by calculating the Root-Sum-Squared (RSS) value:

$$RSS \_ error = \left[ \left( error \# 1 \right)^2 + \left( error \# 2 \right)^2 + \cdots + \left( error \# n \right)^2 \right]^{1/2}$$  \hspace{1cm} (21)

As an example, Table 4 shows the error budget associated with a sRGB to ColorMatch conversion. Of course, if the sRGB R’G’B’ values were previously calculated with a simple gamma, we can remove this contribution from Table 4 and we are left with the Bradford matrix and the R’G’B’ rounding error.

<table>
<thead>
<tr>
<th>Processing steps</th>
<th>Average $\Delta E^*_{ab}$ error</th>
</tr>
</thead>
<tbody>
<tr>
<td>sRGB: R’G’B’ to RGB, simple gamma</td>
<td>1.3</td>
</tr>
<tr>
<td>sRGB to XYZ</td>
<td>0</td>
</tr>
<tr>
<td>Bradford matrix: XYZ to XYZ</td>
<td>1.4</td>
</tr>
<tr>
<td>XYZ to ColorMatch RGB (matrix)</td>
<td>0</td>
</tr>
<tr>
<td>XYZ to ColorMatch RGB (clipping)</td>
<td>Not included (see text)</td>
</tr>
<tr>
<td>ColorMatch: RGB to R’G’B’ rounding</td>
<td>0.23</td>
</tr>
<tr>
<td>Combined RSS error</td>
<td>1.9</td>
</tr>
</tbody>
</table>

Table 4: The error budget associated with a sRGB to ColorMatch conversion. The RGB to R’G’B’ conversion is assumed to be with a simple gamma expression.

<table>
<thead>
<tr>
<th></th>
<th>Apple to ColorMatch</th>
<th>ColorMatch to Apple</th>
<th>Apple to sRGB</th>
<th>sRGB to Apple</th>
<th>sRGB to ColorMatch</th>
<th>sRGB to ColorMatch</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average $\Delta E^*_{ab}$ due to clipping</td>
<td>0.882</td>
<td>2.22</td>
<td>2.81</td>
<td>3.03</td>
<td>4.49</td>
<td>3.52</td>
</tr>
<tr>
<td>Conversions clipped at 0</td>
<td>5.6 %</td>
<td>12.0 %</td>
<td>10.4 %</td>
<td>19.0 %</td>
<td>16.3 %</td>
<td>19.8 %</td>
</tr>
<tr>
<td>Conversions clipped at 1</td>
<td>0.45 %</td>
<td>2.3 %</td>
<td>2.3 %</td>
<td>2.2 %</td>
<td>4.5 %</td>
<td>2.5 %</td>
</tr>
<tr>
<td>Conversions clipped at 0 and 1</td>
<td>0.004 %</td>
<td>0 %</td>
<td>0 %</td>
<td>0.54 %</td>
<td>0 %</td>
<td>0.55 %</td>
</tr>
<tr>
<td>Conversions clipped</td>
<td>6.00 %</td>
<td>14.3 %</td>
<td>12.7 %</td>
<td>20.7 %</td>
<td>20.8 %</td>
<td>21.7 %</td>
</tr>
<tr>
<td>Maximum $\Delta E^*_{ab}$ error</td>
<td>3.7</td>
<td>5.5</td>
<td>8.0</td>
<td>10.9</td>
<td>11.6</td>
<td>13.6</td>
</tr>
<tr>
<td>R’G’B’ coordinates for maximum error</td>
<td>Apple (255, 0, 0)</td>
<td>ColorMatch (0, 255, 255)</td>
<td>Apple (0, 255, 0)</td>
<td>sRGB (0, 255, 0)</td>
<td>ColorMatch (0, 255, 0)</td>
<td>sRGB (0, 255, 0)</td>
</tr>
<tr>
<td>Conversions with $\Delta E^*_{ab}$ &gt; 5</td>
<td>0.00 %</td>
<td>0.21 %</td>
<td>2.03 %</td>
<td>3.90 %</td>
<td>8.27 %</td>
<td>5.53 %</td>
</tr>
<tr>
<td>“xy” surface outside of the destination space</td>
<td>3.20 %</td>
<td>4.54 %</td>
<td>4.00 %</td>
<td>8.75 %</td>
<td>3.05 %</td>
<td>6.56 %</td>
</tr>
<tr>
<td>“u<em>v</em>w” surface outside ...</td>
<td>4.42 %</td>
<td>3.15 %</td>
<td>2.10 %</td>
<td>12.0 %</td>
<td>2.83 %</td>
<td>13.8 %</td>
</tr>
<tr>
<td>L<em>a</em>b* volume outside ...</td>
<td>4.99 %</td>
<td>10.9 %</td>
<td>5.59 %</td>
<td>10.0 %</td>
<td>15.7 %</td>
<td>14.3 %</td>
</tr>
<tr>
<td>L<em>u</em>v* volume outside ...</td>
<td>16.1 %</td>
<td>10.4 %</td>
<td>5.27 %</td>
<td>7.69 %</td>
<td>14.7 %</td>
<td>22.1 %</td>
</tr>
</tbody>
</table>

Table 5: Characteristics of the clipping errors found by converting 100 000 random samples from one space to the other. A conversion is considered clipped when one of the R, G, or B values is clipped.
An average $\Delta E_{ab}$ error of 1.9 can be expected for converting between sRGB and ColorMatch, a result that does not include the effects of clipping which affects only a portion of the conversions.

8. Comparing the four spaces

Table 5 shows some characteristics of the clipping errors found in converting either of the Apple, ColorMatch or sRGB spaces into the other two spaces. From the percentage of conversions clipped, we have a first assessment that the sRGB space is larger than the ColorMatch space, and that the ColorMatch space is larger than the Apple space. The most unexpected result is the high percentage of clipped conversions, up to 21.7% for the sRGB to ColorMatch conversion, a number that cannot be anticipated from the areas of the triangular shapes in the “xy” or “u*v*” chromaticity diagram which are outside of the destination space and which are also shown in Table 5. A similar comparison with the L*a*b* and L*u*v* volumes shows a better match with the clipping behavior but the absolute values are still off.

The largest clipping error occurs for pure blue in the sRGB to ColorMatch conversion, and for pure green in the ColorMatch to sRGB conversion. However, even if the maximum error due to clipping is higher in the sRGB to ColorMatch conversion, there are a higher number of conversions with larger errors when going from ColorMatch to sRGB than from sRGB to ColorMatch. This can be seen in the histogram of Figure 2. The result is a slightly bigger average for the ColorMatch to sRGB conversions.

The higher “visibility” of this effect, and the fact that errors in this space conversion occur in a color range corresponding to many common subjects – sky, vegetation – is certainly one basis of the impression that the ColorMatch space is larger than sRGB. Also, a sRGB to ColorMatch conversion is less likely to create visible color artifacts than the reverse transform.

To help visualize why there are large clipping errors, Figure 3 shows three-dimensional representations of the ColorMatch and sRGB spaces in various color coordinate systems. The initial R'G'B' cube, identical for both spaces, is divided in 125 uniformly sized cubes. The RGB representations of both spaces are then shown and it can be seen how their different file-encoding gammas assign more R'G'B' values to lower luminance colors. The RGB cubes are then transformed into XYZ parallelepipeds where the sRGB volume is apparently much larger than the ColorMatch volume. The size difference is no longer obvious when the sRGB XYZ data is transformed from Illuminant D65 to Illuminant D50 using the simplified Bradford matrix. However, even if they are similar in size and almost coincident in space, it just happens that most of the non-coincident zones are for the densely packed low luminance colors. It can be seen in the XYZ, xyY and L*a*b* diagrams that almost complete slices of each shape – a slice corresponding to 20% of the gamut – are not comprised in the other space. Another view of this mismatch can be seen in Figure 4 where the xyY shapes are flattened into the xy plane. A simplistic comparison of their gamut sizes based on the L*a*b* representation (L*a*b* volume) gives volumes identical within a fraction of a percent.

A better estimate of the relative gamut sizes can be obtained by comparing the transformation between the larger Adobe (1998) space and the three smaller ones. Table 6 shows the conversion statistics of these transforms. The ColorMatch to Adobe transform results are also shown. In this last case, clipping occurs in extremely small regions near the red and blue primaries, which nonetheless corresponds to 0.66% of the ColorMatch gamut.

Figure 2: Histogram of the $\Delta E_{ab}$ errors obtained by converting 100 000 random samples between the ColorMatch and sRGB spaces. Each bin is 0.50 $\Delta E_{ab}$ wide.
Figure 3: A visual comparison of the ColorMatch and sRGB spaces.

$R'G'B'$

ColorMatch D50
sRGB D50

XYZ

L*$a*b*$
Still, the sRGB space exhibits, because of a flatter error distribution, the largest average error. The error histogram is shown in Figure 5.

There again, comparing the clipping values with the results deduced from chromaticity diagram surfaces and space volumes show the inadequacy of these representations to assess clipping.

9. Conclusion

To place the conversion errors in perspective, we have to take into consideration the conditions in which these images will be seen. One of these conditions is the observation time. According to a review article by Has & al., an inexperienced user will take approximately 5 seconds to notice a \( \Delta E_{ab}^* \) difference of 15 from an original. The time goes up to 10 seconds for a \( \Delta E_{ab}^* \) of 10, and 15 seconds for a \( \Delta E_{ab}^* \) of 5. Another study has shown that errors of less than 2,5 \( \Delta E_{ab}^* \) are not visible on real world images shown on a CRT. In essence, the threshold value of \( \Delta E_{ab}^* = 1 \) can only be achieved only by prolonged comparative viewing in a controlled environment.

On the hardware side, it has been shown that CRTs require a warm-up time varying between 15 minutes and three hours, depending on models, before achieving a long term stability of 0,15 \( \Delta E_{ab}^* \) on average. On a given CRT subjected to a large luminance variation, an initial \( \Delta E_{ab}^* \) of 1,0 was seen to exponentially decrease to about 0,1 \( \Delta E_{ab}^* \) in 60 seconds. As for printed material, errors between 2 and 4 \( \Delta E_{ab}^* \) are mentioned by Has & al. for the offset and rotogravure process.

<table>
<thead>
<tr>
<th></th>
<th>Adobe to Apple</th>
<th>Adobe to ColorMatch</th>
<th>ColorMatch to Adobe</th>
<th>Adobe to sRGB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average ( \Delta E_{ab}^* ) due to clipping</td>
<td>10,7</td>
<td>9,93</td>
<td>0,960</td>
<td>14,0</td>
</tr>
<tr>
<td>Conversions clipped at 0</td>
<td>49,5 %</td>
<td>47,7 %</td>
<td>0,65 %</td>
<td>35,7 %</td>
</tr>
<tr>
<td>Conversions clipped at 1</td>
<td>11,4 %</td>
<td>9,8 %</td>
<td>0,011 %</td>
<td>10,9 %</td>
</tr>
<tr>
<td>Conversions clipped at 0 and 1</td>
<td>5,36 %</td>
<td>4,9 %</td>
<td>0 %</td>
<td>1,2 %</td>
</tr>
<tr>
<td>Conversions clipped</td>
<td>55,6 %</td>
<td>52,6 %</td>
<td>0,66 %</td>
<td>45,3 %</td>
</tr>
<tr>
<td>Maximum ( \Delta E_{ab}^* ) error</td>
<td>46,7</td>
<td>43,4</td>
<td>2,3</td>
<td>50,4</td>
</tr>
<tr>
<td>R’G’B’ coordinates for maximum error</td>
<td>Adobe (0, 255, 0)</td>
<td>Adobe (0, 255, 0)</td>
<td>ColorMatch (0, 0, 255)</td>
<td>Adobe (0, 255, 0)</td>
</tr>
<tr>
<td><strong>xy</strong> surface outside of the destination space</td>
<td>29,5 %</td>
<td>28,6 %</td>
<td>0,011 %</td>
<td>25,9 %</td>
</tr>
<tr>
<td><strong>u<em>v</em>+</strong> surface outside …</td>
<td>22,9 %</td>
<td>24,0 %</td>
<td>0,024 %</td>
<td>14,3 %</td>
</tr>
<tr>
<td>L<em>a</em>b* volume outside …</td>
<td>35,0 %</td>
<td>32,3 %</td>
<td>0,011 %</td>
<td>31,9 %</td>
</tr>
<tr>
<td>L<em>u</em>v* volume outside …</td>
<td>28,8 %</td>
<td>31,6 %</td>
<td>0,011 %</td>
<td>27,0 %</td>
</tr>
</tbody>
</table>

Table 6: Characteristics of the clipping errors found by converting 100 000 random samples from the large Adobe (1998) space to three smaller spaces. The Apple and sRGB spaces are completely enclosed by the Adobe space. The ColorMatch space has a small portion outside of the Adobe space and its conversion statistics towards the Adobe space are shown.
With these numbers as a baseline, the 1.9 \( \Delta E_{ab} \) conversion error of Table 4, for conversions where there is no clipping, will not be perceived in most cases. The same is true for clipped conversions where the average error due to clipping is less than 5 \( \Delta E_{ab} \), a reasonable limit for most non-critical work. Nonetheless, the maximum error due to clipping is sufficiently high in many cases to be readily perceived, especially if the image contains large zones of a single out-of-gamut color.

Some conversions are preferable to others. For example, based on a 5 \( \Delta E_{ab} \) limit — the amount of conversions clipped with more than 5 \( \Delta E_{ab} \) — are shown in Table 5 — the conversions between Apple RGB and ColorMatch are not likely to result in noticeable differences. The same is true, but with a lesser extent for the Apple RGB to sRGB conversion where only 2% of the conversions have an error between 5 and 8 \( \Delta E_{ab} \). Even though the clipping errors can be large, it is preferable to convert from sRGB to ColorMatch than the reverse since 25% (≈ 5.53% / 21.7%) of the conversions are higher than 5 \( \Delta E_{ab} \) for the first case, and 40% in the later. From this point of view the ColorMatch is the largest of the three gamuts.

From the point of view of the clipping statistics, the sRGB is the space which appears to have the largest gamut, either when looking at statistics between the three similar size spaces or between the larger Adobe (1998) space and these three spaces. An intuitive argument which goes in line with this conclusion is that the red-blue area in the chromaticity diagram, where the sRGB space extends more than ColorMatch, is a region where smaller \( xy \) increments are required for just perceived differences (MacAdam ellipses) than for the blue-green region where the ColorMatch extends farther than sRGB.

What these two point of view show is that relative gamut size cannot be determined on the number of clipped conversions alone or on the maximum conversion error. How the clipping errors are perceived is crucial as well as how a given color space covers colors which are important for the end application.

Finally, this comparison has shown that both the b-dimensional chromaticity diagram and the \( L^*a^*b^* \) or \( L^*u^*v^* \) volumes cannot be used, as they are often done, to accurately justify gamut sizes or out-of-gamut proportions.
References

   “Frequently Asked Questions About Gamma,” available as “GammaFAQ.pdf”;
   “The rehabilitation of gamma,” available as “Rehabilitation_of_gamma.pdf”;
   “Frequently Asked Questions About Color,” available as “ColorFAQ.pdf”.
16. From Reference 11. The gamma function of that reference is defined with a different equation than the one used in this document. The gamma curve of the reference was regenerated and fitted against the detailed gamma function defined in this document.
22. The method is described in Reference 20. The results are from the slide presentation at the conference.

Document revised 2003-10-06.

About the author

Danny Pascale, M.Sc.A., B.Eng.: Following a Bachelor degree in Engineering Physics at École Polytechnique, University of Montreal, he obtained his Master’s degree in the same group for the study of non-linear effects in optical fibers. Subsequently, he participated in the development of lasers and instrumentation dedicated to laser-matter interaction research at INRS-Energie. He then designed electro-optical instrumentation and thermal sensors for military applications at Bendix Avelux, a unit of Allied-Signal Aerospace. A few years later he became a partner at Simdev Electronics, a military applications at Bendix Avelex, a unit of Allied-Signal Aerospace. From there he took charge of video display and positioning systems development. He now does technology assessment and helps companies bring new products into the market in the computer and consumer electronics industries. He recently formed a new company, BabelColor, dedicated to the development of colorimetric software tools. He is a member of SPIE, OSA, and IEEE.